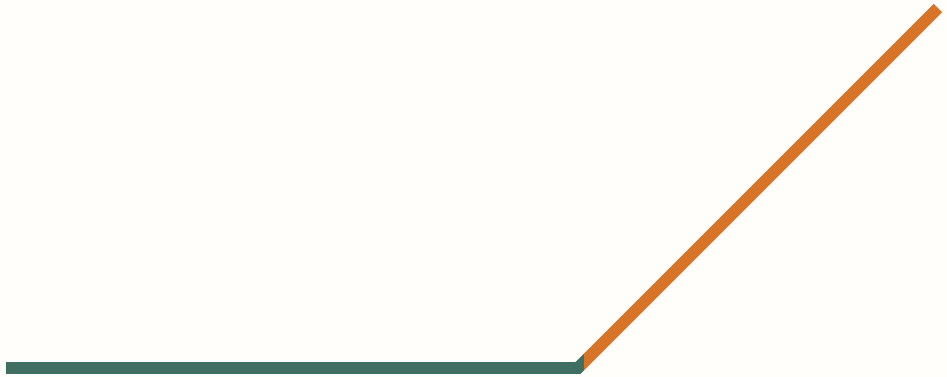
Volatility Curves and Surfaces

From a noisy options board to a picture of risk across strikes and time.

09



Event signatures in options markets?

The case of the 2024 U.S. election

Presidential elections are often high-volatility events. The date is known in advance, but the result and the market's reaction are not.

OCTOBER 31

Cboe analysis

What were traders pricing before the vote?

NOVEMBER 5

Election day

The date of the event was known in advance.

NOVEMBER 6

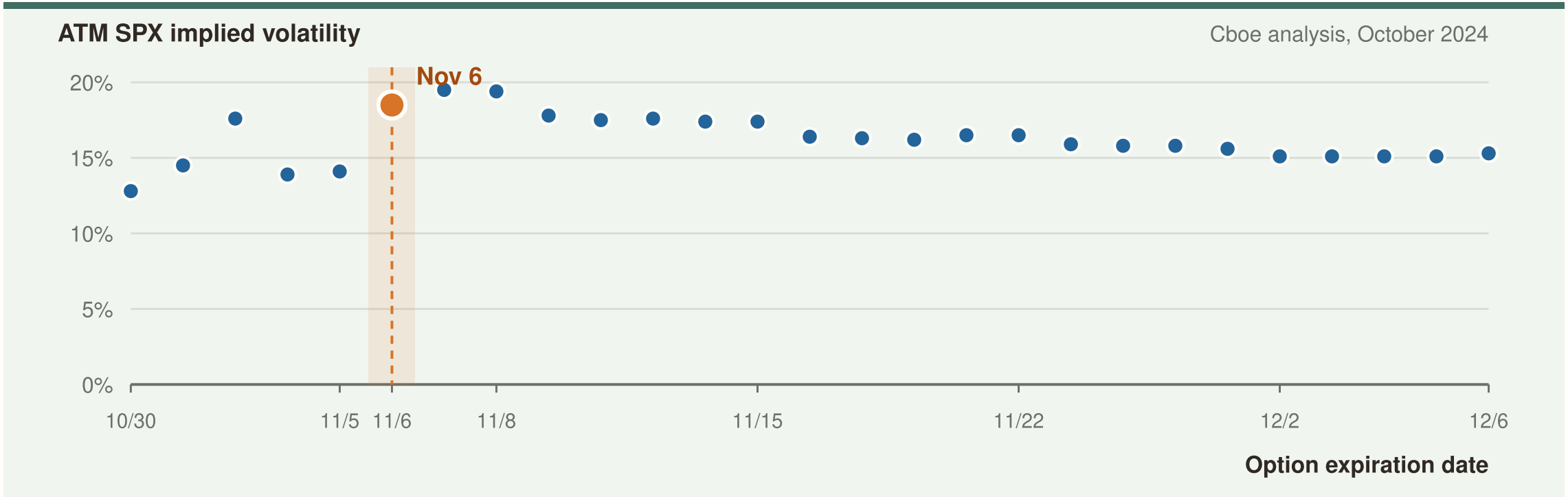
Results expected

The first expiry likely to include the result.

Source: Cboe Global Markets, October 31, 2024.

An election bump in implied volatility

At-the-money SPX implied volatility, plotted by option expiration date.

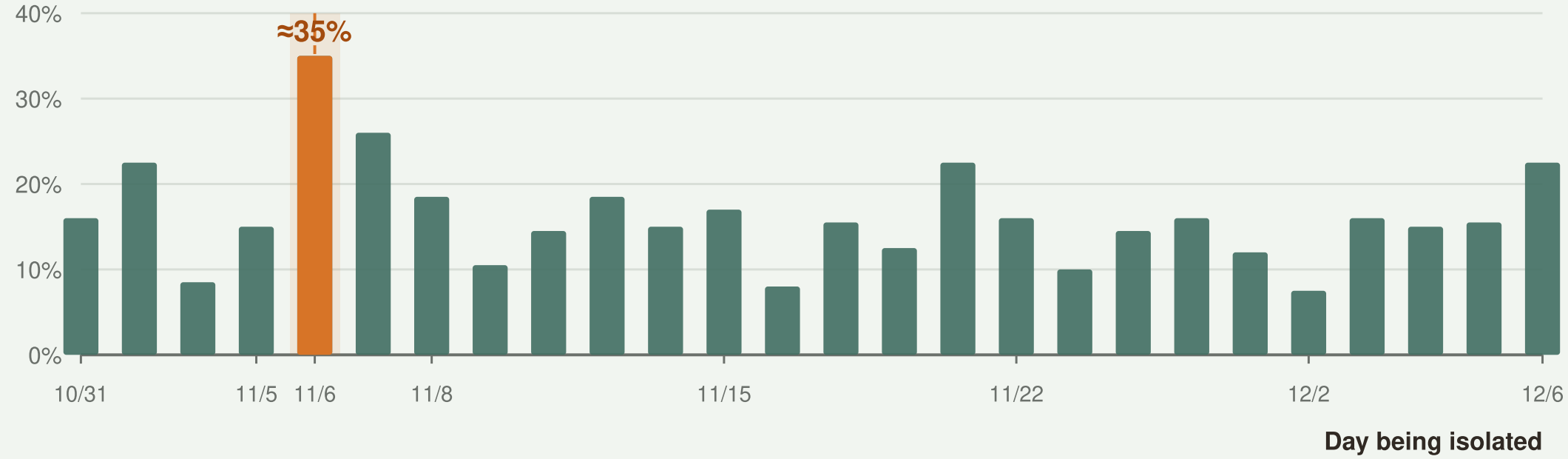


The first expirations spanning election night are richer. But each dot covers **all the days up to that expiry**.

Approximate values redrawn from Cboe Global Markets, October 2024.

Isolate the day itself

Cboe isolated the extra uncertainty attributed to each day. This is **forward volatility**.

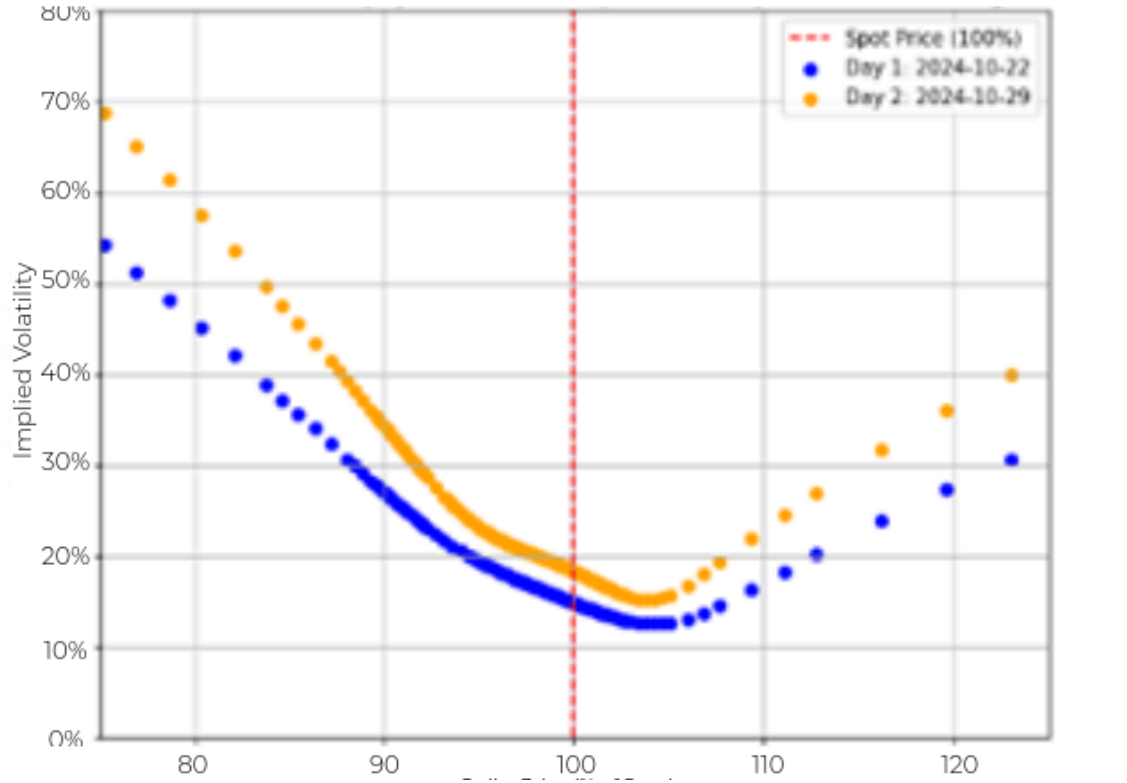


November 6 stands out at roughly **35% annualized**. That is about 2.2% over one trading day, not a 35% daily move.

Approximate values redrawn from Cboe Global Markets, October 2024. We will derive the forward-volatility calculation later in this lecture.

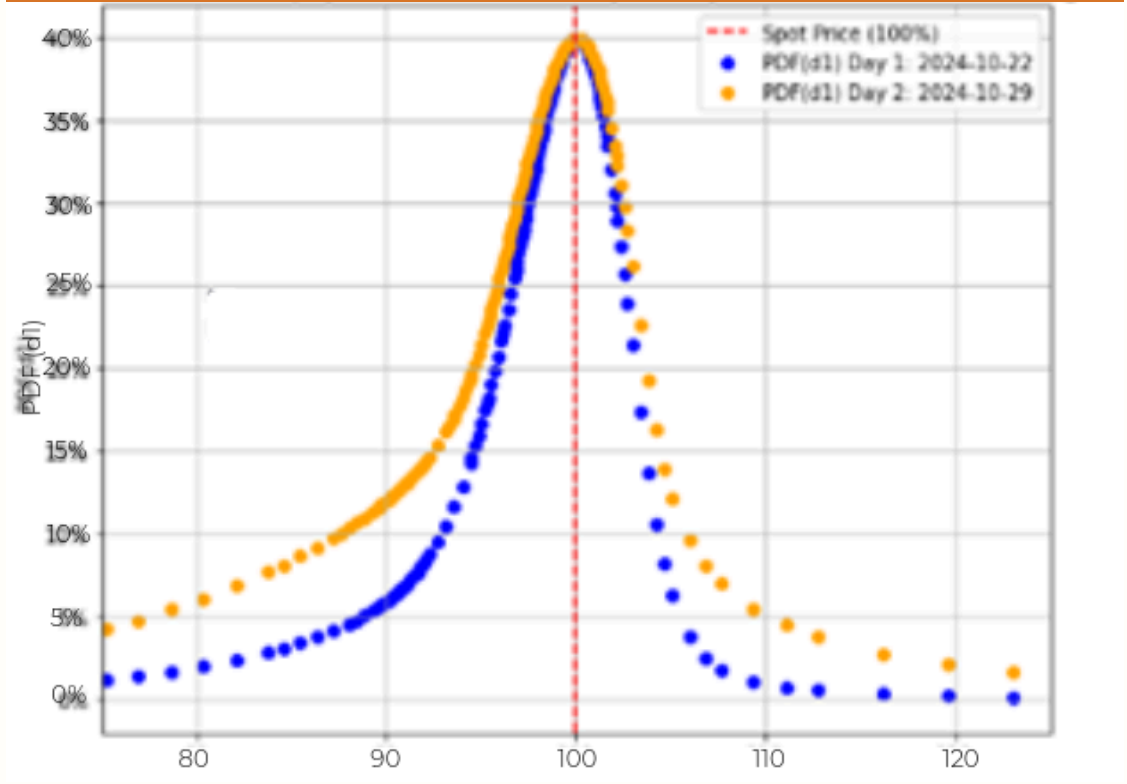
A strike curve and an implied distribution

Implied volatility by strike



Strike as % of spot

Unnormalized PDF



Strike as % of spot

Can we generate these plots ourselves?

Source: Cboe Global Markets, October 31, 2024. Original figure, showing its two strike-percentage panels.

How can we figure this out from raw market data?

A real options board does not arrive as a neat volatility curve. We have three jobs.

1

Get the data

Collect bid and ask quotes across strikes and expirations, plus the underlying price.

→

2

Clean the data

Reject empty or stale markets. Find the forward with parity and prefer useful OTM quotes.

→

3

Interpret the information

Fit curves, compare expirations, and separate genuine structure from quote noise.

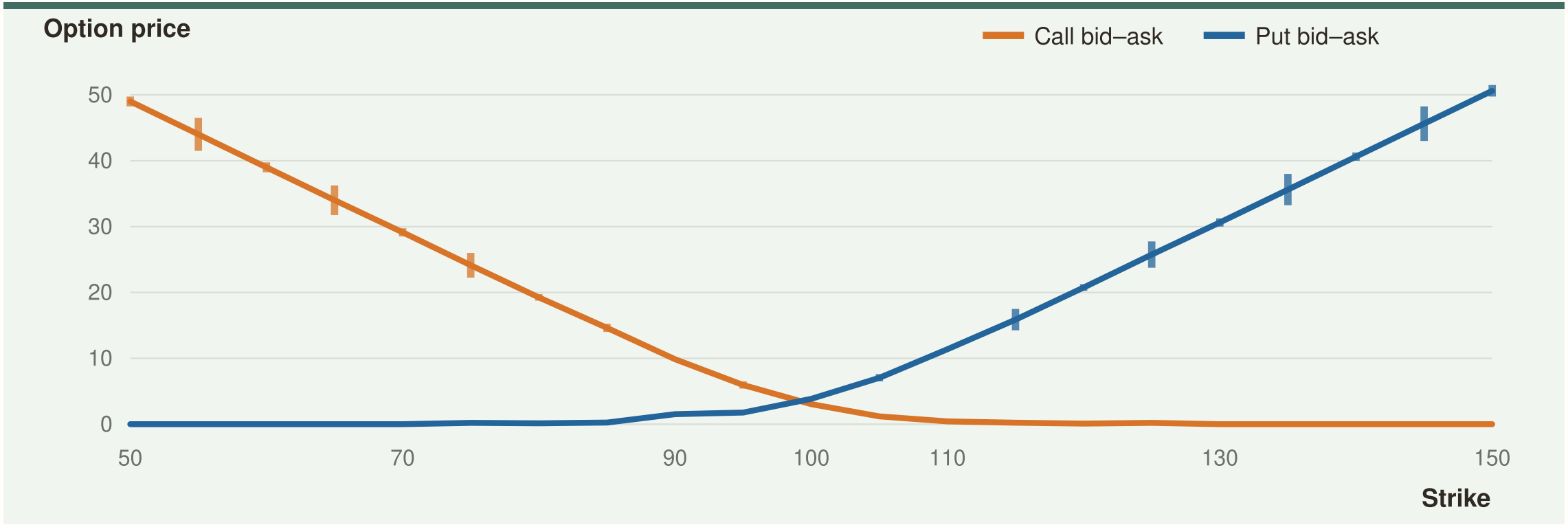
Start with the options board

One stock at $100. One-month European options at 21 strikes. A synthetic board.

CALLS					STRIKE	PUTS				
SIZE	BID		ASK	SIZE	K	SIZE	BID		ASK	SIZE
(2)	48.25	–	49.75	(2)	50					
(1)	41.50	–	46.50	(1)	55					
(2)	38.25	–	39.75	(2)	60					
(1)	31.75	–	36.25	(1)	65					
(5)	28.50	–	29.75	(5)	70					
(3)	22.25	–	26.00	(3)	75	(3)	0.05	–	0.35	(3)
(5)	18.75	–	19.75	(5)	80	(5)	0.05	–	0.20	(5)
(3)	14.00	–	15.25	(3)	85	(3)	0.05	–	0.45	(3)
(12)	9.70	–	10.00	(12)	90	(12)	1.45	–	1.60	(12)
(3)	5.40	–	6.50	(3)	95	(3)	1.60	–	1.95	(3)
(12)	3.00	–	3.10	(12)	100	(12)	3.70	–	4.00	(12)
(3)	1.00	–	1.35	(3)	105	(3)	6.50	–	7.60	(3)
(12)	0.35	–	0.50	(12)	110	(12)	11.25	–	11.50	(12)
(3)	0.05	–	0.40	(3)	115	(3)	14.25	–	17.50	(3)
(5)	0.05	–	0.15	(5)	120	(5)	20.25	–	21.25	(5)
(3)	0.05	–	0.35	(3)	125	(3)	23.75	–	27.75	(3)
					130	(5)	30.00	–	31.25	(5)
					135	(1)	33.25	–	38.00	(1)
					140	(2)	40.00	–	41.25	(2)
					145	(1)	43.00	–	48.25	(1)
					150	(2)	49.75	–	51.50	(2)

First, graph the prices

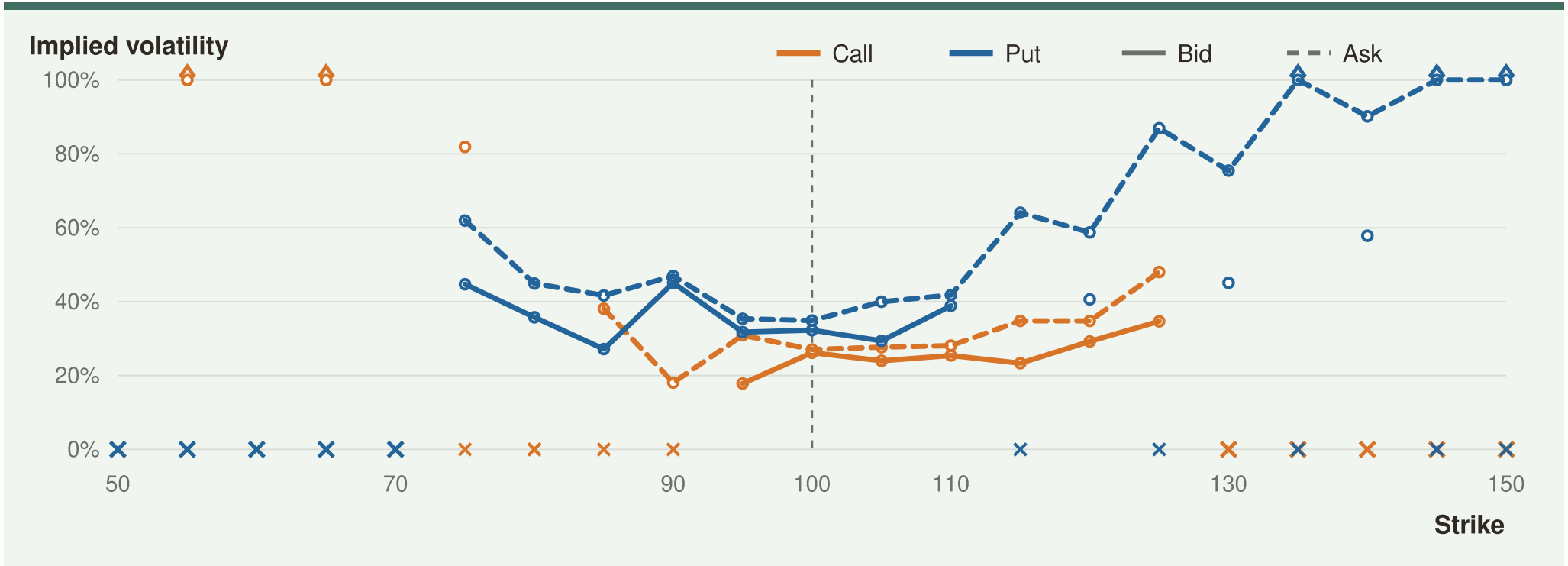
Every vertical mark is a bid-to-ask range. The expensive in-the-money options set the scale.



This is not a useful picture of how expensive the *optional part* is. The strikes are barely comparable.

Convert every price into implied volatility

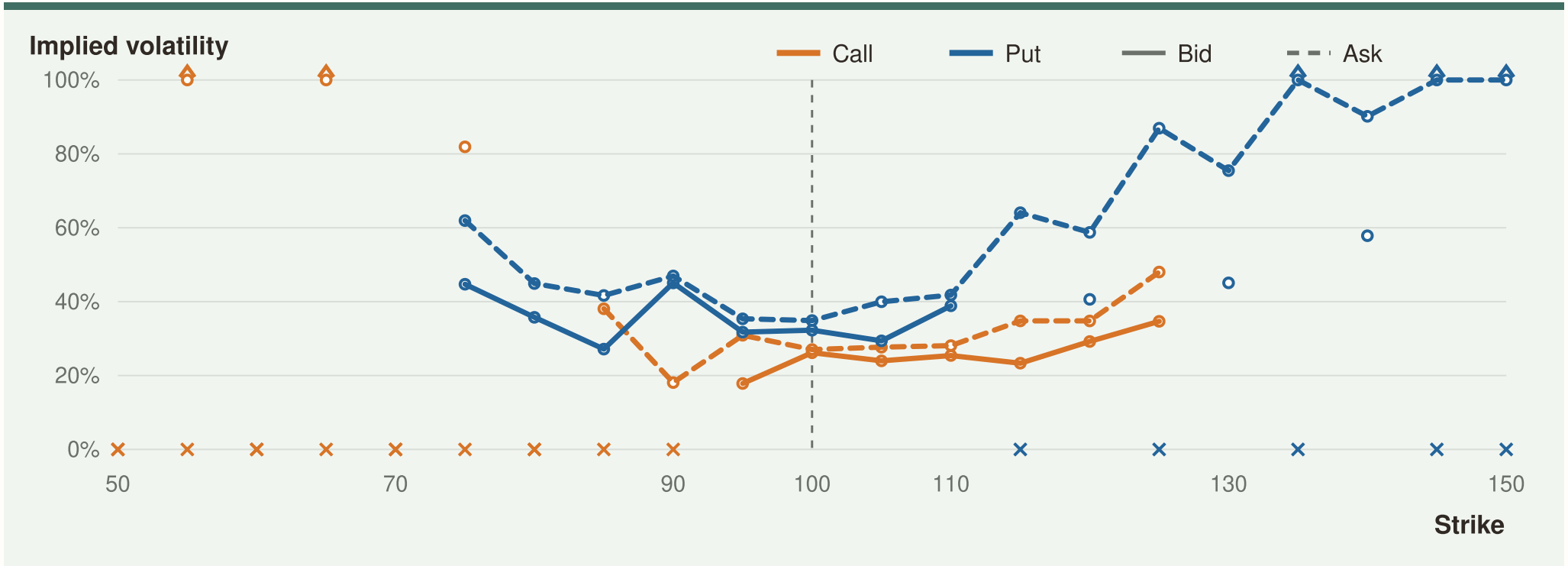
Invert each bid and ask with Black-Scholes, using $100 as the forward. Crosses mean missing or invalid quotes; arrows mean above 100%.



The common scale helps, but this is a mess. Let us fix **one problem at a time**.

Problem 1: blank quotes are not markets

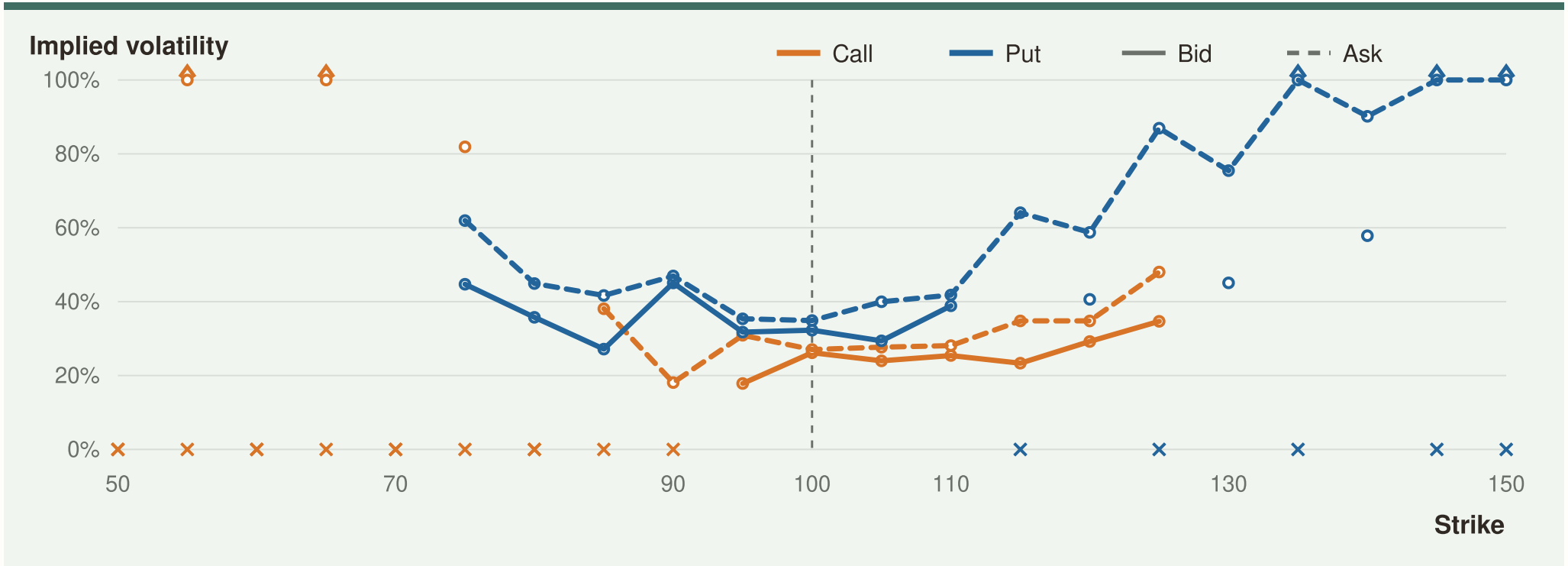
Some wing strikes have neither a bid nor an ask. No one is quoting them. Delete those observations.



The blank listings are gone. The call and put curves still disagree.

Problem 2: calls and puts disagree

At the same strike and expiration, European put-call parity says a call and put have the **same implied volatility**. Why are the blue put quotes systematically above the orange calls?

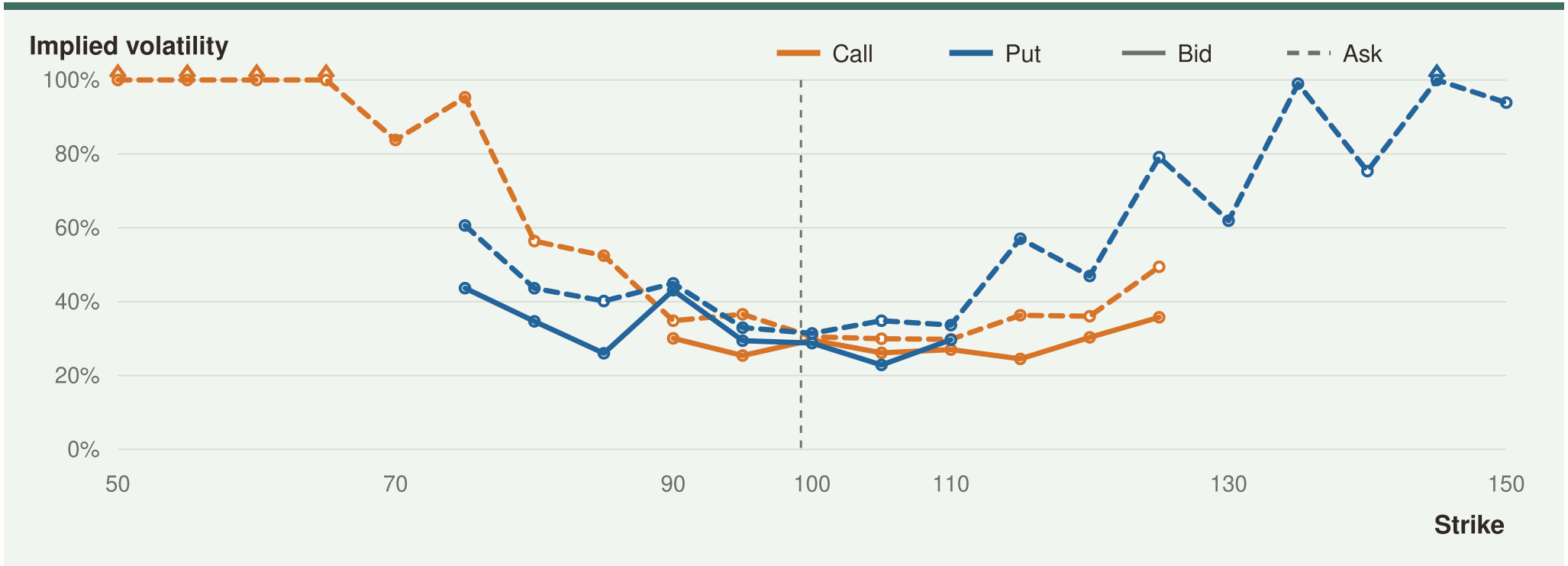


We used $100 as the forward. But funding and dividends can move the forward away from spot.

Correct the forward with parity

$$C - P = D(F - K)$$

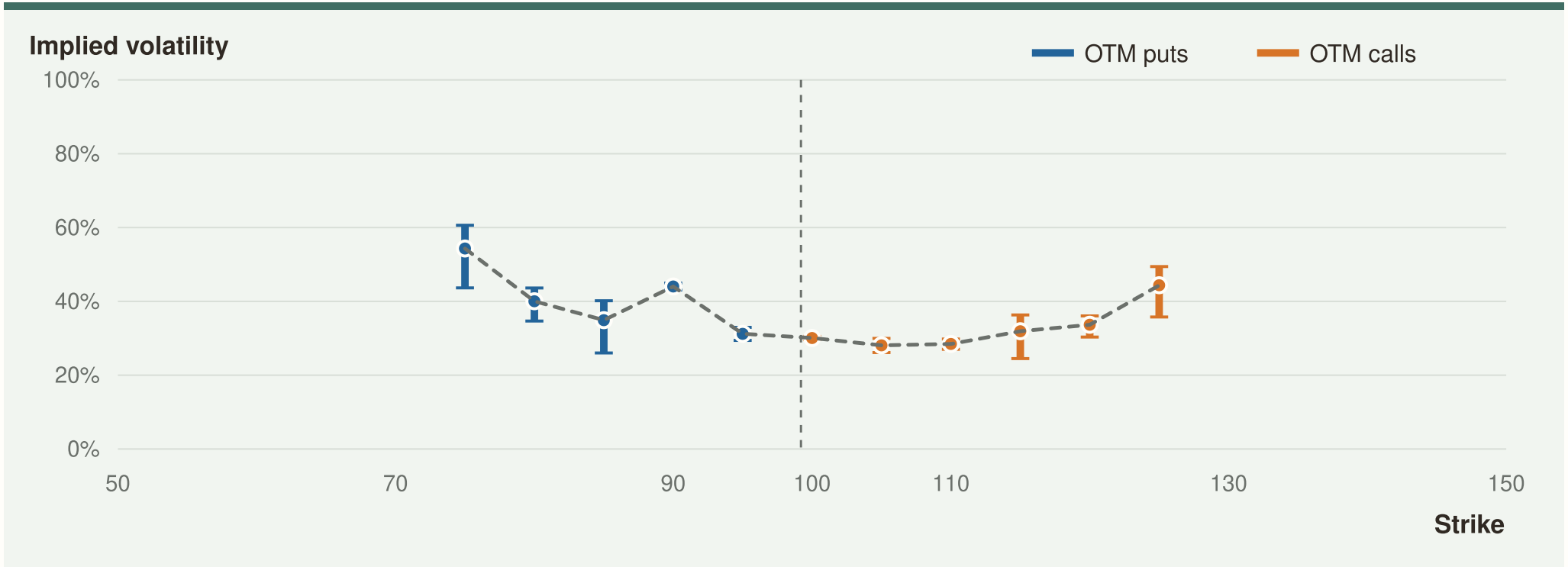
At the 100 strike, the two midpoints imply $F \approx 99.2$.



Calls and puts now agree near the money. The in-the-money wings still look awful, and the 90-put is suspicious.

Problem 3: which side should we use?

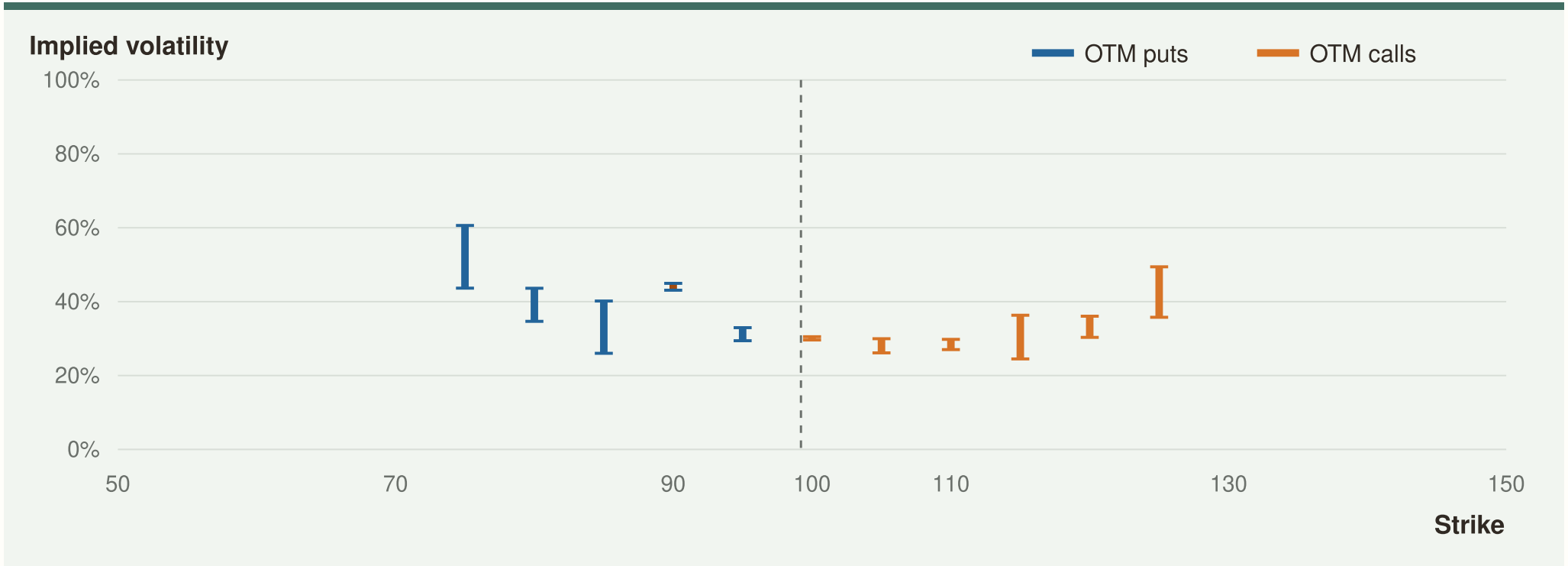
On the left, out-of-the-money puts are cleaner. On the right, out-of-the-money calls are cleaner.



They avoid a large intrinsic-value component and are usually the more liquid way to trade the same option risk.

Problem 4: mids exaggerate the bumps

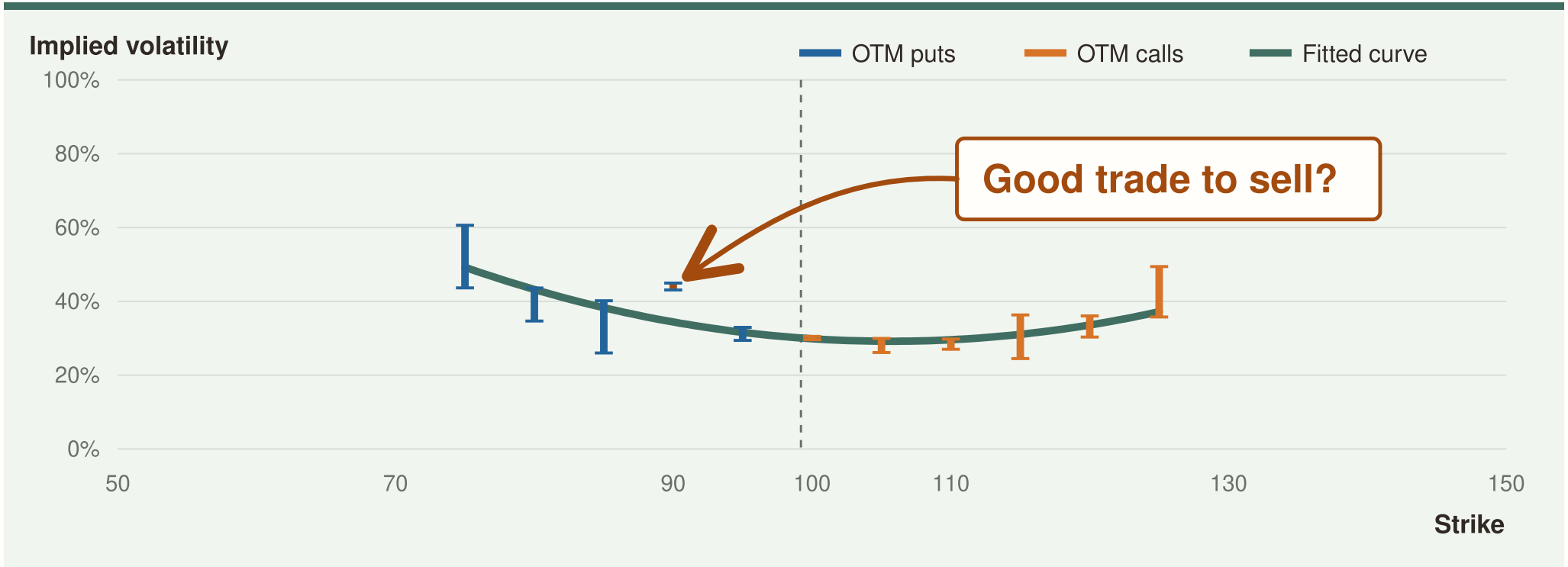
Each price is a **bid and ask**. In volatility space, that is a vertical interval, not one perfect midpoint.



Two nearby midpoint dots can look different while their actual tradable ranges overlap.

Fit a curve through the useful markets

A simple smooth fit gives more weight to narrow markets and excludes the stale 90-put.



The curve is a modeling choice. It is not a magical “true volatility” hidden behind the board.

What did we actually do?

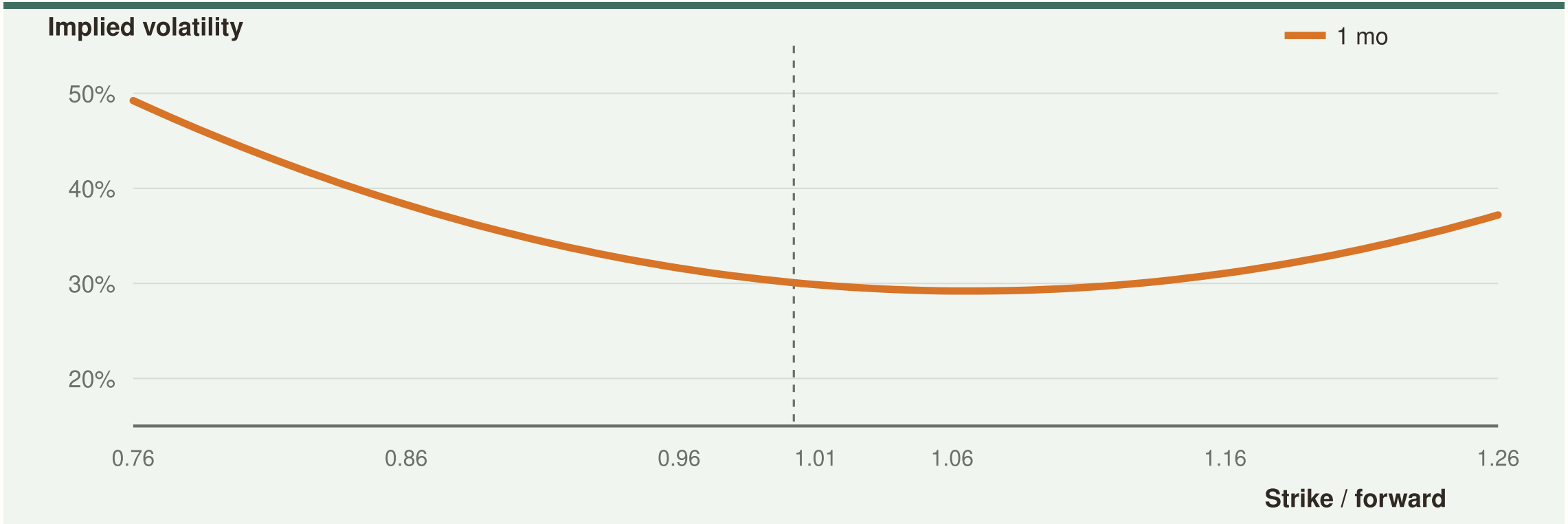
1. Turn the same raw prices into call and put volatility bid-ask ranges.
2. Remove listed strikes with no bid and no ask.
3. Use parity to locate the forward and line up calls with puts.
4. Prefer the more informative out-of-the-money side.
5. Respect quote widths, investigate outliers, then fit a smooth curve.

Pause



One expiration gives us one curve

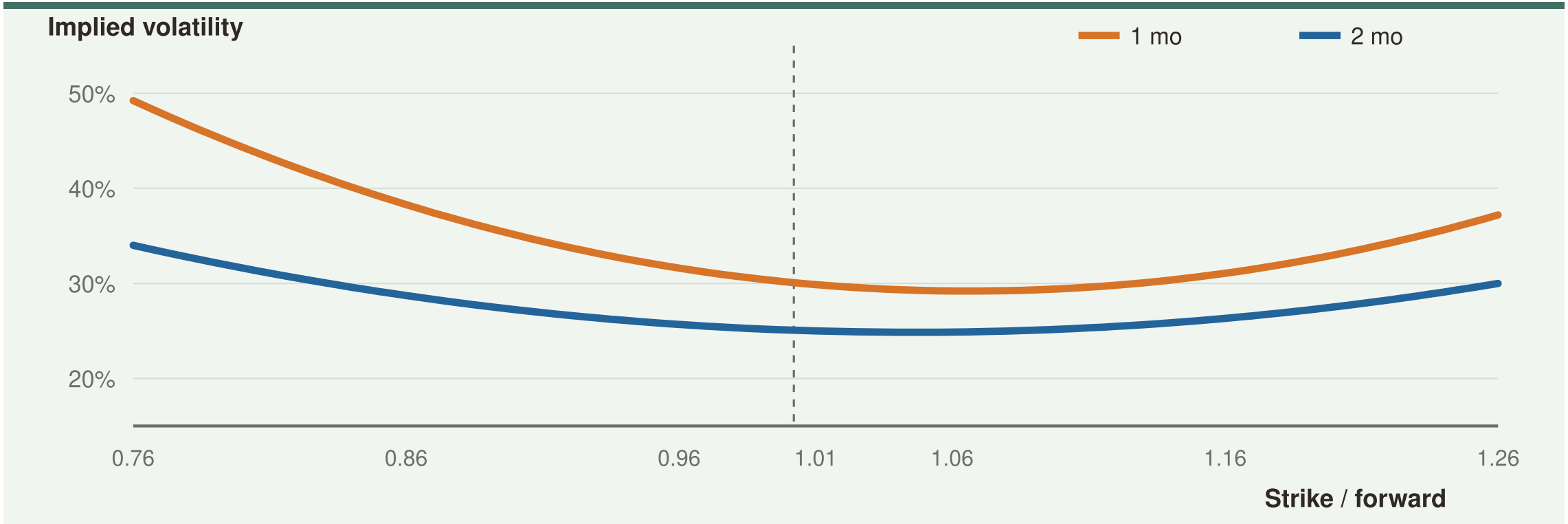
This is our cleaned one-month implied volatility curve.



The horizontal axis is $K/F(T)$, so “at the money” means the strike is near that expiration’s forward.

Now add a second expiration

The two-month options have their own curve. Their volatility numbers include **both** months, not just the second.



If one-month ATM volatility is 30% and two-month ATM volatility is 25%, what is the market implying for *month two alone*?

Volatility adds in variance-time

The variance accumulated up to expiration T is the quantity we can subtract.

$$w(T) = \sigma_{\text{imp}}(T)^2 T$$

Month 1	Month 2
$w(T_1)$	$w(T_2) - w(T_1)$

The second block is **forward variance**, the extra variance attributed to that interval.

Extract the forward volatility

Let $\sigma_{1\to 2}$ be the annualized volatility *during* the interval from T_1 to T_2.

Variance in the interval

$$w(T_2) - w(T_1) = \sigma_{1\to 2}^2 (T_2 - T_1)$$

Substitute $w(T_i) = \sigma_i^2 T_i$

$$\sigma_2^2 T_2 - \sigma_1^2 T_1 = \sigma_{1\to 2}^2 (T_2 - T_1)$$

Divide and take the positive root

$$\sigma_{1\to 2} = \sqrt{\frac{\sigma_2^2 T_2 - \sigma_1^2 T_1}{T_2 - T_1}}$$

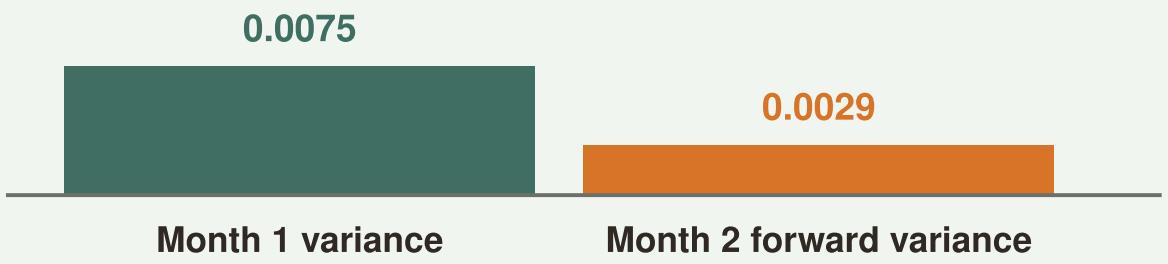
One month at 30%. Two months at 25%.

$$\sigma_{1\to 2} = \sqrt{\frac{0.25^2(2/12) - 0.30^2(1/12)}{1/12}} \approx 18.7\%$$

The later period can be quiet even though the two-month option still carries the volatile first month.

The forward is sensitive to both inputs

Keep the first month at 30%. Move the two-month quote and watch the implied second-month volatility.



Two-month implied vol

25.0%

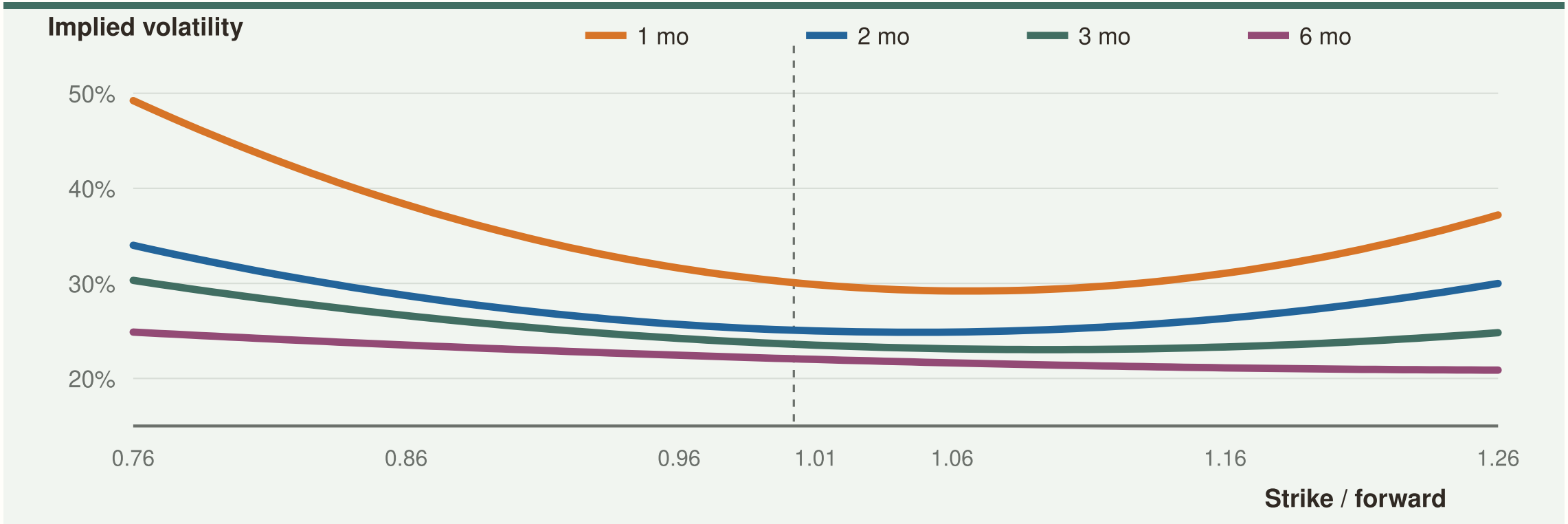
Month-two forward vol

18.7%

A later-expiry volatility below an earlier one is not automatically an arbitrage. The accumulated variance must still rise.

Four expirations make a family of curves

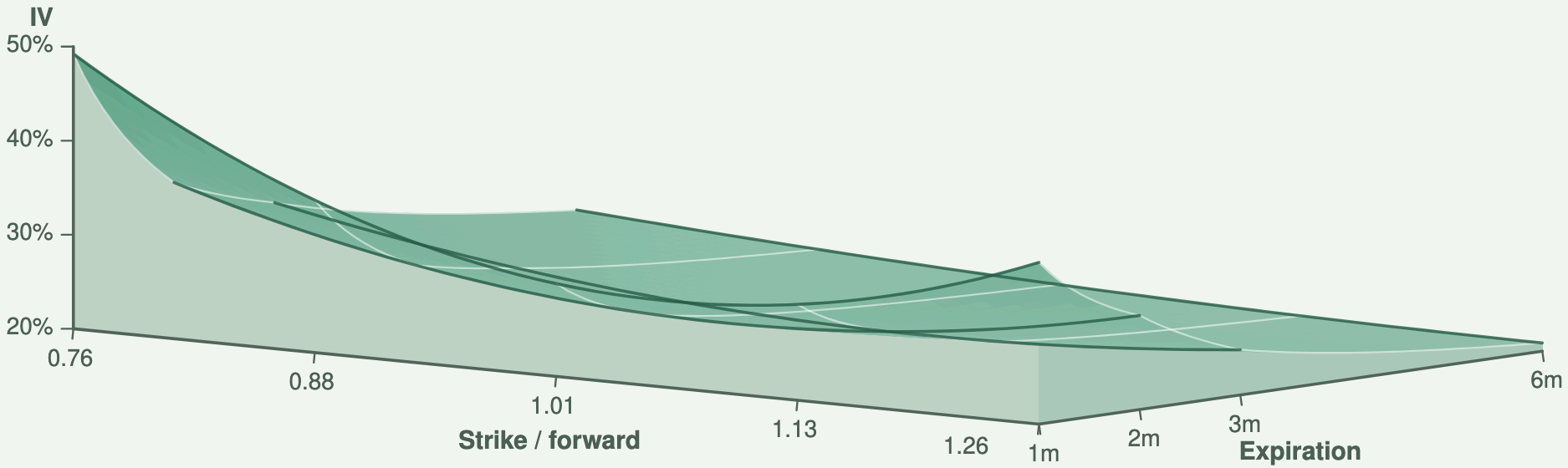
Add three and six months. Each expiration has its own strike curve.



This is a collection of slices through one object: the **volatility surface**.

The whole surface

The base axes are strike relative to the forward and time to expiration. Height is implied volatility.



A surface summarizes *both* skew across strikes and term structure across expirations.

Event risk changes the term structure

An announcement contributes uncertainty only to options that expire **after** it.



BEFORE THE EVENT

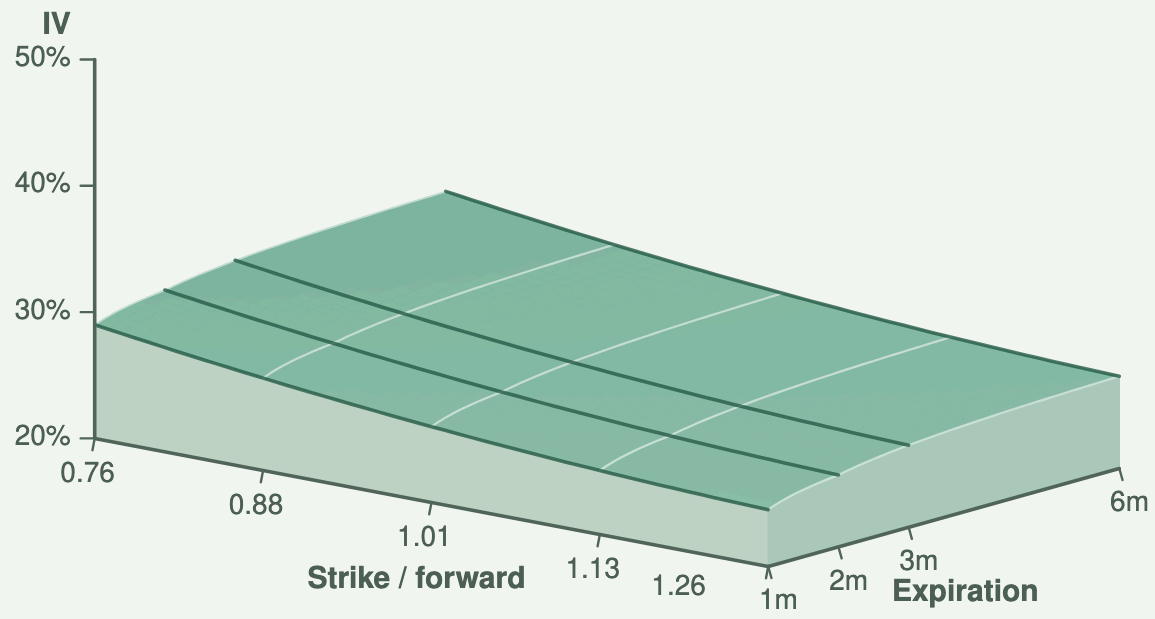
The one-month option expires first. It cannot contain the announcement jump.

AFTER THE EVENT

The two-month option spans the jump. Its extra variance is concentrated in the forward interval.

At longer expirations the same one-time event is averaged over more time, so its effect on *annualized* IV is diluted.

Case 1: an ordinary stock



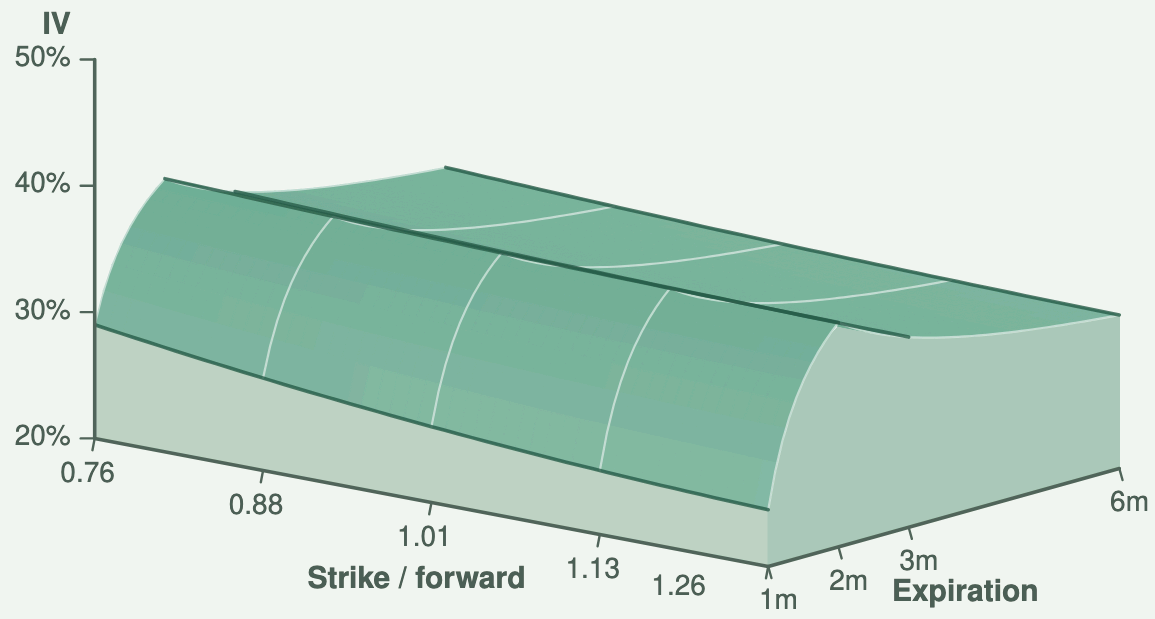
WHAT STANDS OUT?

A smooth downside smirk. At every strike, IV rises slightly with expiration.

POSSIBLE INTERPRETATION

Ordinary business risk plus a market preference for downside protection.

Case 2: earnings in six weeks



WHAT STANDS OUT?

One month expires before earnings. Two months includes the event, raising IV across strikes. The skew often flattens.

LOOK CAREFULLY

Three- and six-month IV can be lower than two-month IV even while their *total variance* is higher.

Case 3: a biotech FDA decision



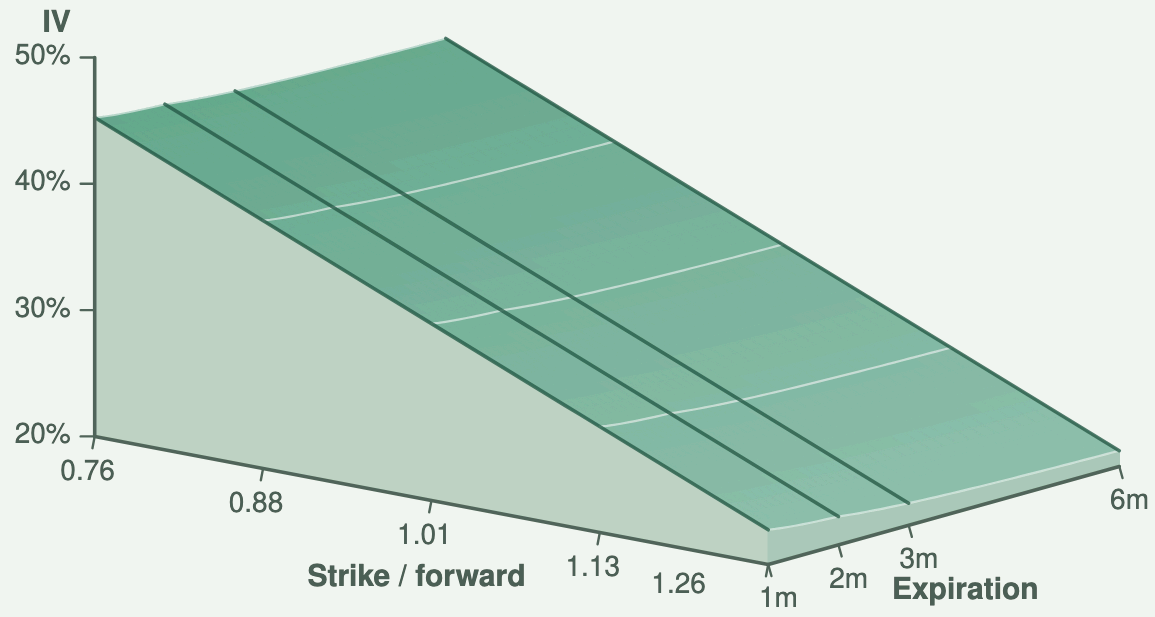
WHAT STANDS OUT?

The one- and two-month smiles are mild. The decision makes the three-month strike curve much more curved.

SIX MONTHS

The smile remains more curved than before the event, but the one-time jump matters less per year.

Case 4: a highly levered stock



WHAT STANDS OUT?

The downside wing is much richer at every expiration. The steep skew is smooth across strikes.

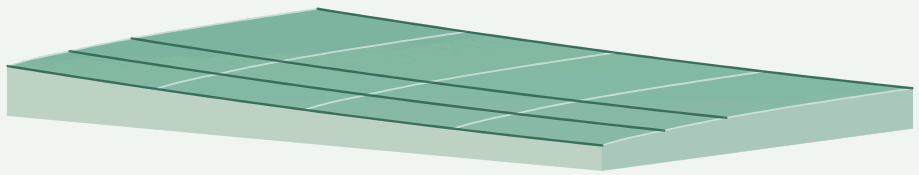
WHY LEVERAGE MATTERS

Equity is the residual after debt. When company value falls, the same debt sits on a smaller equity cushion.

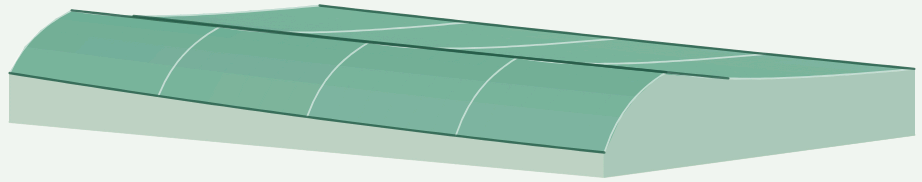
Which part of the surface changed?

Same IV scale and viewing angle across all four.

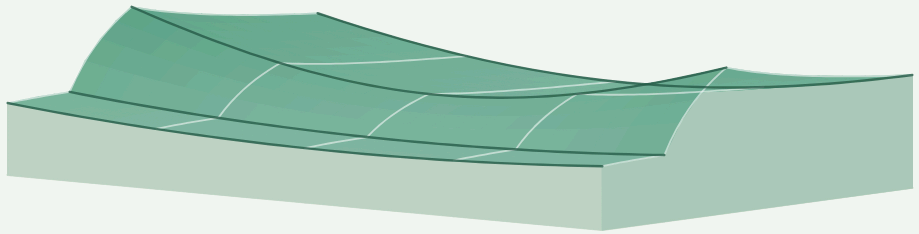
Ordinary



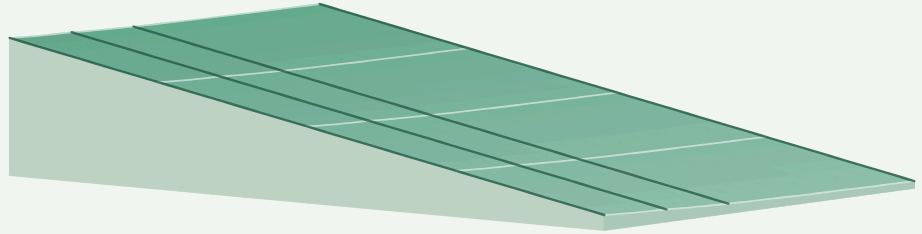
Earnings



Biotech



Levered



Ask two separate questions: **where across strikes?** and **which forward interval?**

What should we believe?

1. A quote can be missing, wide, or stale. Do not mistake those artifacts for market beliefs.
2. Parity and the forward must be right before call and put vols can be compared.
3. A fitted surface should be smooth enough to be useful, but flexible enough to preserve genuine event structure.
4. A surface describes option prices. It is not, by itself, a forecast of real-world outcomes.

Summary

1. Clean the board before fitting anything.
2. Use parity to locate the forward and OTM quotes to read the wings.
3. Bid and ask imply volatility ranges, not one perfect number.
4. Forward volatility isolates an interval by subtracting total variance.
5. A surface has strike and expiration dimensions. Events and leverage change them differently.

Next: Lecture 10 asks what no-arbitrage and option prices imply about possible outcomes.