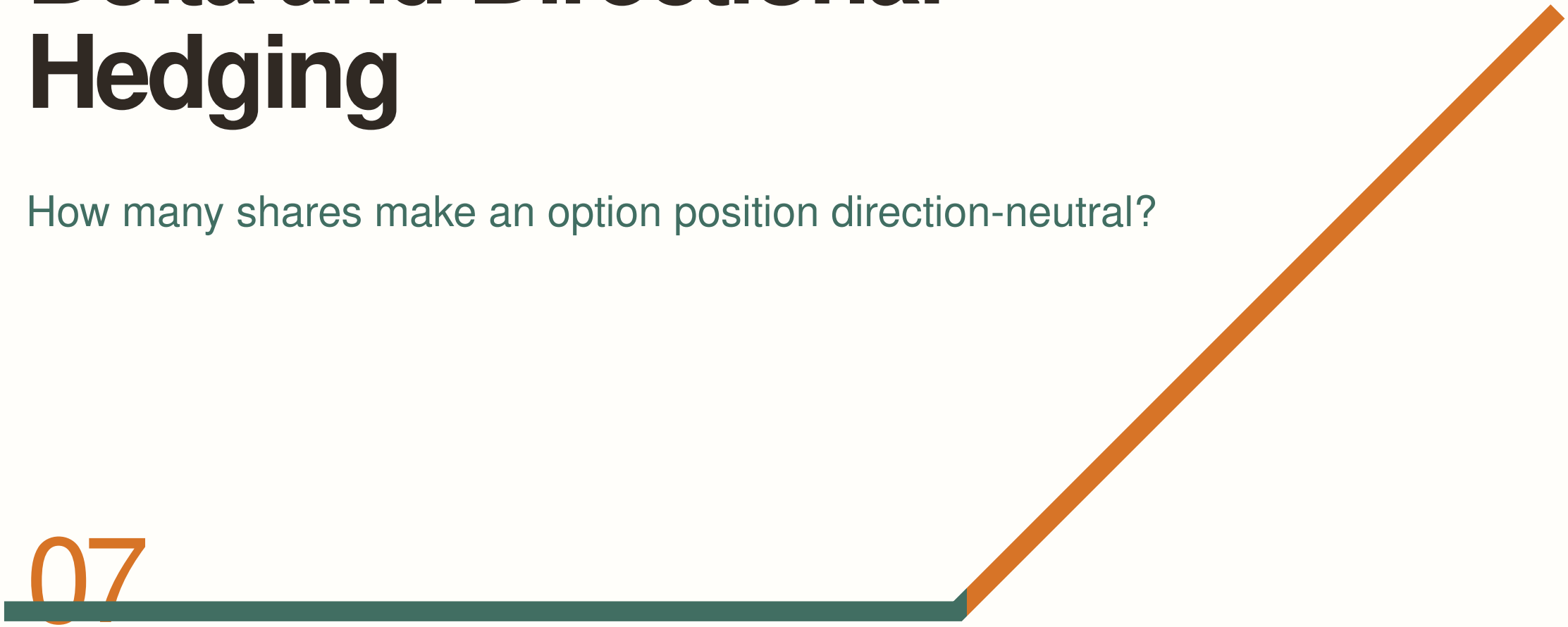
Delta and Directional Hedging

How many shares make an option position direction-neutral?

07



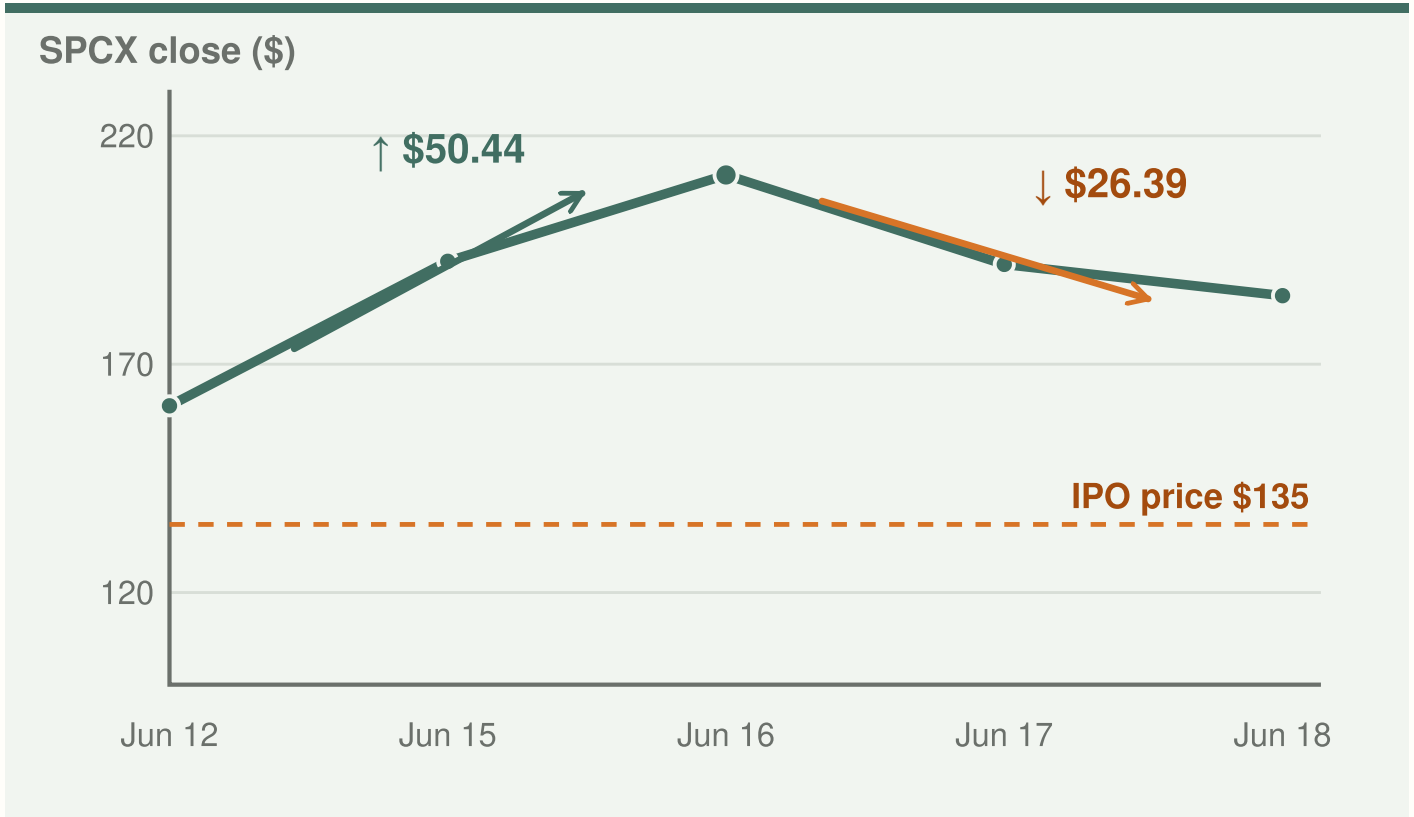
Real-world options and hedging

SPACEX IPO · JUNE 2026

Imagine you hold **10,000 deep-in-the-money employee calls** with a $20 strike. Your position is locked up. You cannot exercise or sell it yet.

At the first close of **$160.95**, intrinsic value is about **$1.41 million on paper**. Delta is close to **+10,000 shares**.

How can you protect that exposure?



The obvious hedge: sell stock

You cannot sell your locked-up grant. Could you **borrow freely tradable shares and sell them short** instead?

EMPLOYEE CALLS

About **+10,000 shares** of delta

OPEN-MARKET SHORT SALE

Borrow and sell **10,000 shares**. Delta **–10,000**

COMBINED POSITION

Approximately **zero delta** for a small stock move

But check the lock-up. SpaceX's IPO filing describes restrictions on *short sales and hedging* of locked-up equity. The economically obvious hedge may be prohibited.

What happened after the first week?

SPCX daily closes from the IPO through **November 5, 2026**.



If you cannot hedge, your directional exposure stays with you until the restriction lifts.

Why hedge?

Most of us are risk averse. If a hedge removes a large source of variance at modest cost, the more predictable outcome can be worth more to us.

The SpaceX position: 10,000 deep-in-the-money calls

STOCK SENSITIVITY

+10,000 shares of delta

A $1 stock move changes the position by about $10,000

CURRENT VALUE

About $1.41 million

Intrinsic value at $160.95 per share and a $20 strike

If permitted, hedging by selling stock reduces your sensitivity to SpaceX and also protects (reduces variance of) the actualized value of the position.

Enron: many employees held company stock in their 401(k) (retirement savings) accounts. When the company failed, they lost jobs *and* substantial retirement savings. The risks were concentrated in the same company.

Hedging can actualize a volatility mispricing

Suppose you sell an option at **22% implied volatility** because you expect the stock to realize **20%**. How do you trade that view without making a large directional bet?

SELL AND HOLD

Keep the premium, then wait for expiry. Stock direction can swamp your volatility edge. The profit varies enormously across price paths.

SELL AND DELTA-HEDGE

Offset the option's stock exposure, then keep adjusting the hedge. Much of the directional noise is removed, so the volatility gap matters more.

In the ideal continuous-hedging model, if volatility really stays at 20%, the 22% sale earns a positive edge **path by path**. Real markets add gaps, hedge error, and trading costs, so profit is not guaranteed.

We will cover this in detail in Lecture 08.

How many shares offset a call?

The SpaceX calls were deep in the money, so each behaved almost like one share. That made the share count easy. What about an at-the-money call?

DEEP IN THE MONEY

Long 1 call → about **+1 share** of stock exposure → sell about **1 share** to hedge

AT THE MONEY

Long 1 call → **?** shares of stock exposure → sell **?** shares to hedge

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To hedge an option, we need its **share-equivalent** for a small stock move. Where can we get that number?

Lecture 05 already gave us that number

Start with the Black–Scholes call price. Only the stock price S changes.

$$\Delta_C = \frac{\partial C}{\partial S}$$

$$\boxed{C(S) = SN(d_1) - Ke^{-rT}N(d_2)}$$

$$d_1 = \frac{\ln(S/K) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, \qquad d_2 = d_1 - \sigma\sqrt{T}$$

1. Both d terms depend on S

$$\frac{\partial d_1}{\partial S} = \frac{\partial d_2}{\partial S} = \frac{1}{S\sigma\sqrt{T}}$$

2. Apply the product rule and chain rule. $\phi = N'$

$$\frac{\partial C}{\partial S} = N(d_1) + S\phi(d_1)\frac{\partial d_1}{\partial S} - Ke^{-rT}\phi(d_2)\frac{\partial d_2}{\partial S}$$

Why do the extra terms cancel?

The two density terms look different. Use the definitions of d_1 and d_2 to compare them.

3. Compare the normal densities

$$d_1 - d_2 = \sigma\sqrt{T}, \qquad d_1 + d_2 = \frac{2\left[\ln(S/K) + rT\right]}{\sigma\sqrt{T}}$$

$$\frac{\phi(d_1)}{\phi(d_2)} = e^{-(d_1-d_2)(d_1+d_2)/2}$$

$$= e^{-\ln(S/K)-rT} = \frac{Ke^{-rT}}{S}$$

$$\boxed{S\phi(d_1) = Ke^{-rT}\phi(d_2)}$$

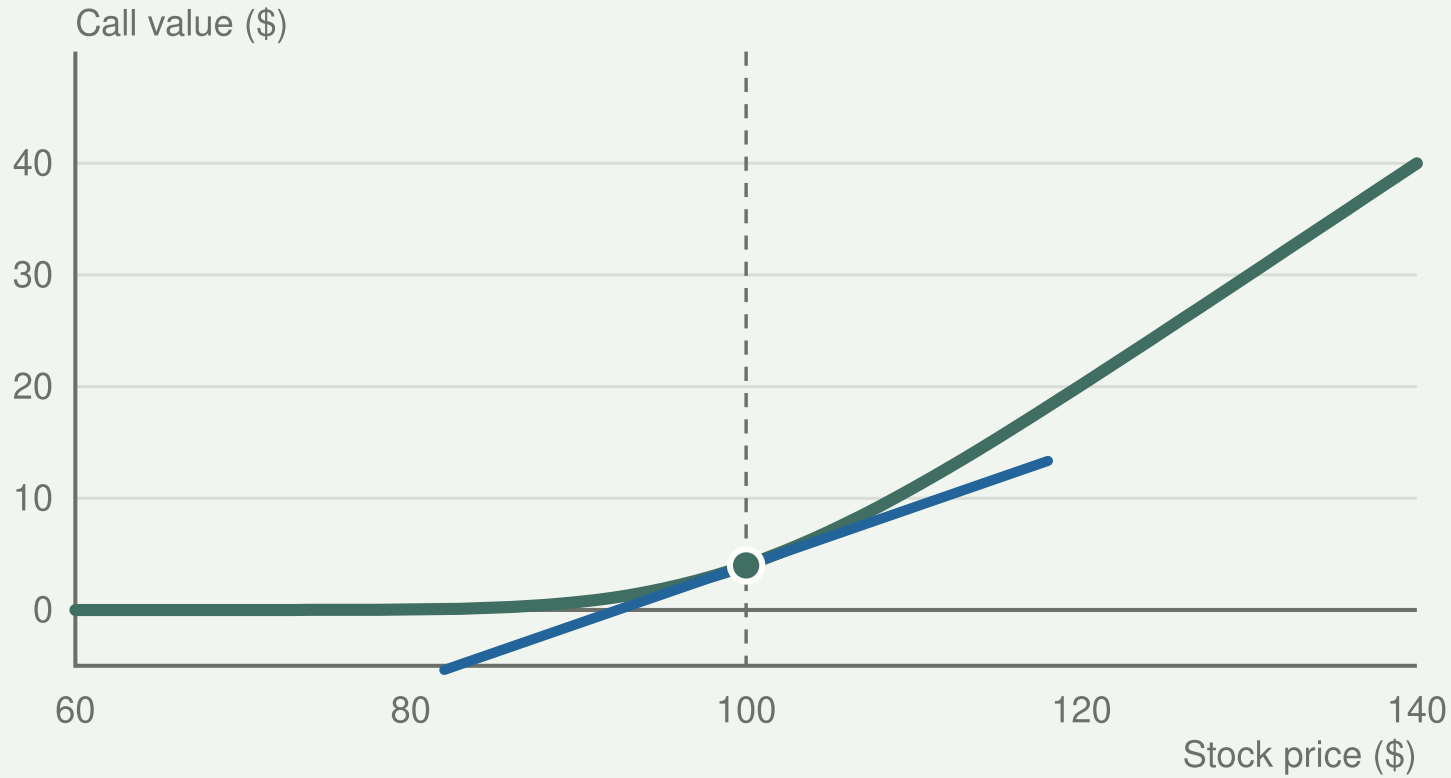
4. Substitute the derivatives from the previous slide

$$\frac{\partial C}{\partial S} = N(d_1) + \frac{S\phi(d_1) - Ke^{-rT}\phi(d_2)}{S\sigma\sqrt{T}}$$

$$= \boxed{N(d_1)}$$

The slope tells us how many shares

In our at-the-money example, $d_1 = 0.05$, so delta is $N(d_1) \approx 0.52$. Move the stock price to see the slope and the required stock hedge change.



AT THIS STOCK PRICE

Stock	**$100**
Call delta	**0.520 share**
Call value	**$3.99**

Green: call value. Blue: local tangent.

Add stock hedge

Stock price ($)

70 130

From the binomial slope to delta

Lecture 04 matched two possible stock outcomes with one stock-and-cash line.

$$\Delta_{\text{tree}} = \frac{C_u - C_d}{S_u - S_d}$$

As the possible moves become tiny:

$$\boxed{\Delta_C = \frac{\partial C}{\partial S}}$$

Delta is the **local slope** of the option-price curve. One unit of delta means one share of directional exposure per option.

Read the units carefully

One call

$\Delta_C \approx 0.52$ share per call.

A $1 stock move

Call value changes by about $\$0.52$.

Ten contracts

At 100 shares per contract, the position has about 520 shares of delta.

Delta is **not** a 52% return on the option. The approximation is for a small stock move with the other inputs held fixed.

Parity gives us the put delta

Lecture 03 gave us $C - P = S - Ke^{-rT}$. Hold K, r, and T fixed and change only S.

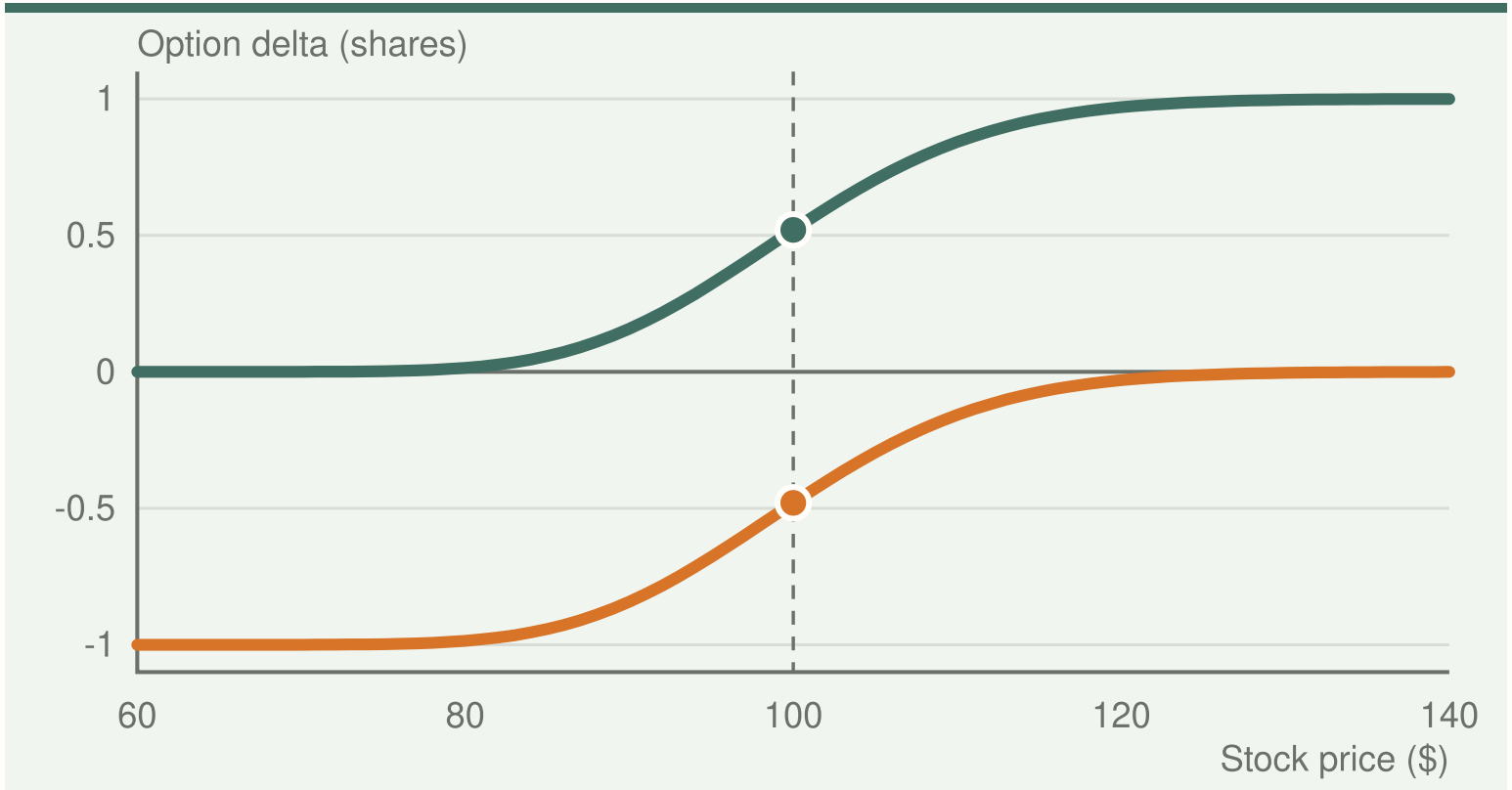
$$\frac{\partial C}{\partial S} - \frac{\partial P}{\partial S} = 1$$

$$\boxed{\Delta_P = \Delta_C - 1}$$

At our at-the-money example: call delta $\approx +0.52$, put delta ≈ -0.48. Their difference is exactly one share.

Call and put exposure change with stock

The same strike and expiry, but opposite directional exposure. Move the common stock-price marker.



SAME STOCK AND STRIKE

Stock	**$100**
Call delta	**+0.520**
Put delta	**-0.480**
Difference	**1.000 share**

The curves stay exactly one share apart.

Stock price ($)

70 130

Pause



Delta is not the chance of exercise

The two normal-distribution terms in Lecture 05 do different jobs.

CALL DELTA

$$N(d_1) \approx 0.520$$

The stock holding in the replicating portfolio.

EXERCISE PROBABILITY UNDER Q

$$N(d_2) \approx 0.480$$

The model probability that $S_T > K$.

They are close in this example, but they are not the same quantity. A 52-delta call does not mean a 52% exercise probability.

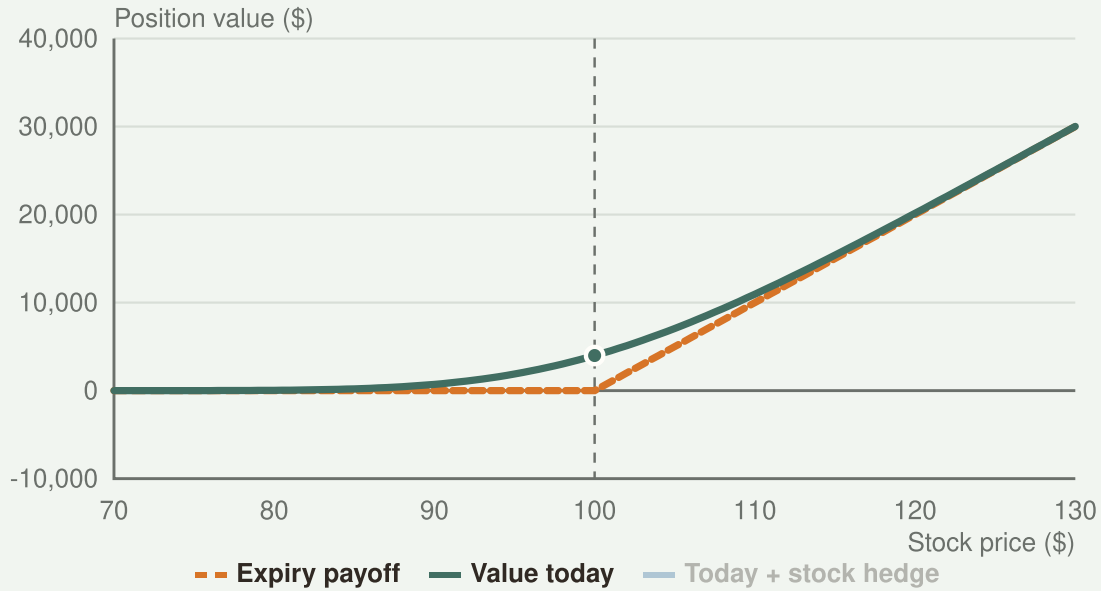
Hedge a long call

You own 10 calls, each on 100 shares. Each call has delta $+0.52$.

Option position	$10 \times 100 \times 0.52 = +520$ **shares**
Stock hedge	**Short 520 shares**
Net delta	$520 - 520 = 0$ **shares**

SAME POSITION, THREE VIEWS

Stock hedge: off



At each stock price, 3 months and 20% vol remain fixed. The stock hedge is set at $100, then held fixed.

Near $S = \$100$, a $\$1$ stock move changes the calls by about $\$520$ and the short stock by $-\$520$.

Hedge a long put

Now you own 10 matching puts. Each put has delta -0.48.

Option position	$10 \times 100 \times (-0.48) = -480$ **shares**
Stock hedge	**Buy 480 shares**
Net delta	$-480 + 480 = 0$ **shares**



The hedge direction flips because the put gains value when stock falls.

Portfolio delta is just a sum

What stock trade makes this book direction-neutral right now?

Long 10 calls at $+0.52$	$+520$ **shares**
Short 4 puts at -0.48	$+192$ **shares**
Long 100 shares of stock	$+100$ **shares**
Net before hedge	$+812$ **shares**



Short 812 shares to reach zero delta. Shorting a negative-delta put **adds** positive delta.

Something obvious

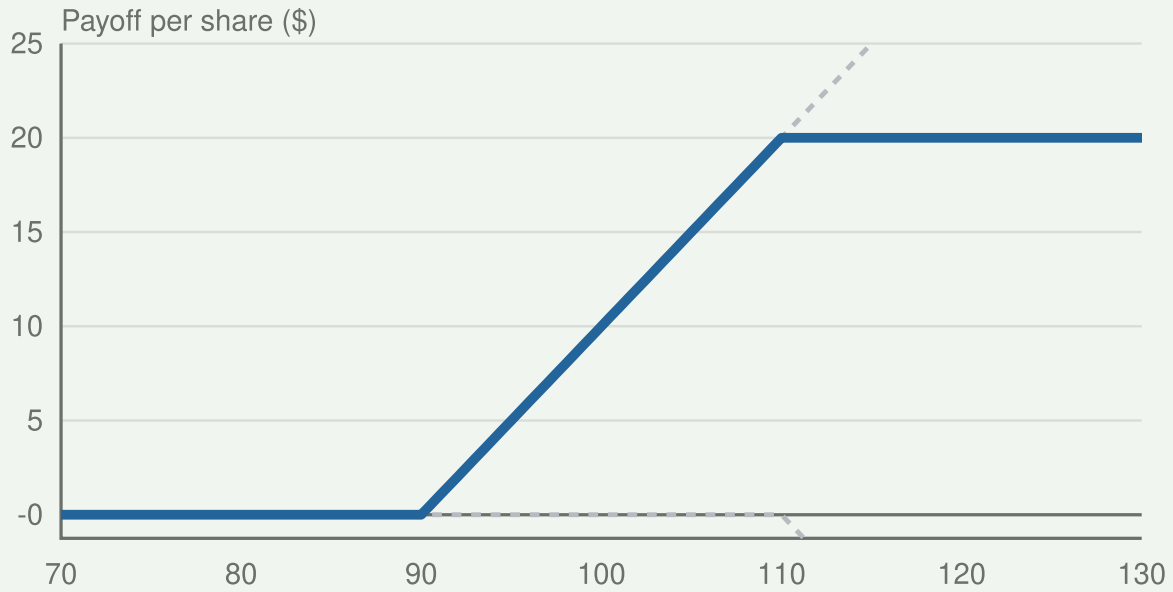
An option portfolio is literally the sum of its legs. A name does not create an edge.

Call spread | Put spread | Butterfly | Condor | Broken-wing fly

Call spread =

+call	**K = $90**
–call	**K = $110**

Add the legs point by point.

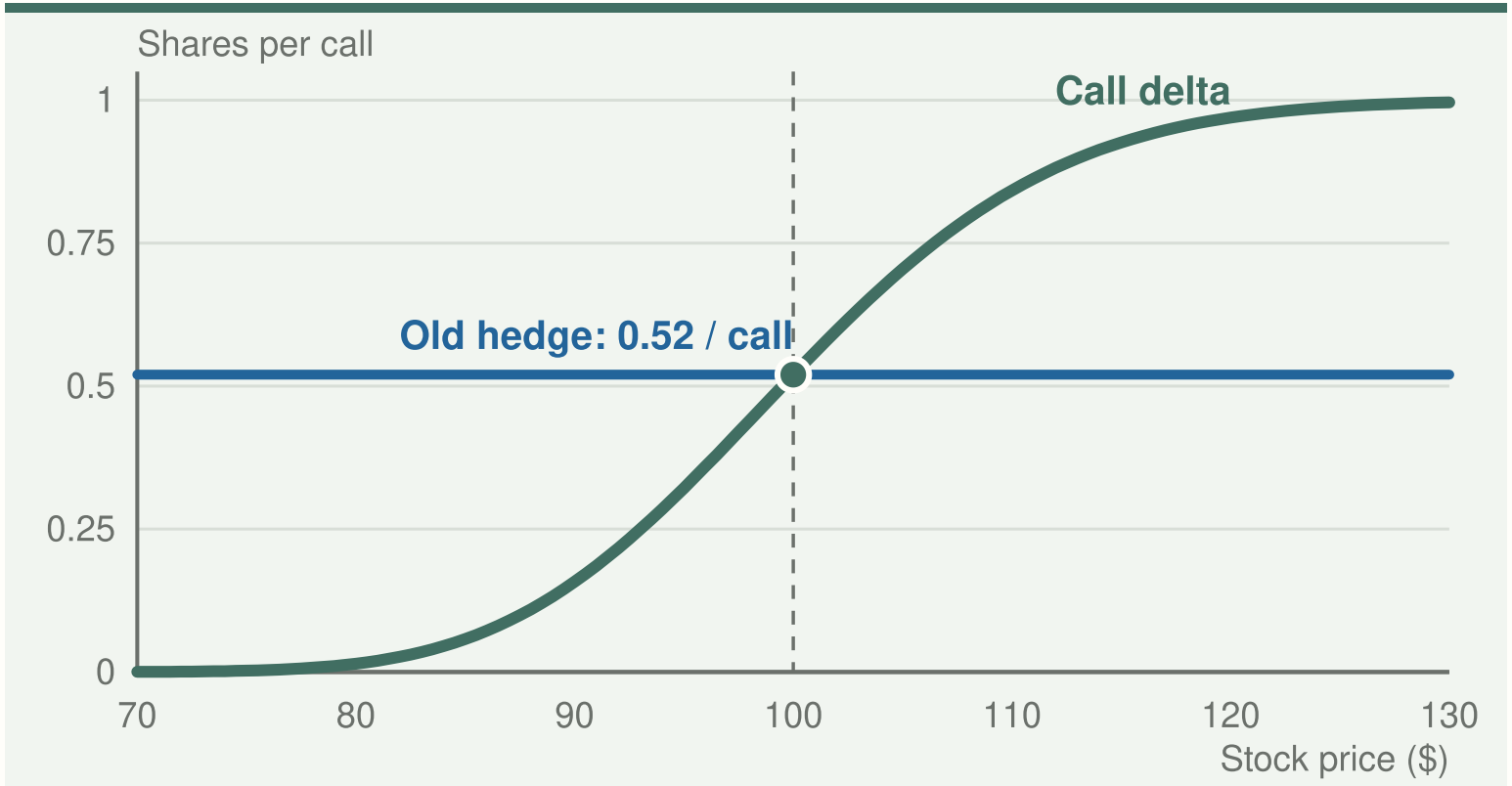


Expiry payoff shown before premiums. Profit also includes what you paid or received for each leg.

$\Pi_{\text{portfolio}} = \sum_i \Pi_i$. If every leg loses in an outcome, the portfolio loses. If every leg has negative expected P&L under your forecast, so does their sum. Structures can change **risk shape**, not manufacture profit.

What happens when stock moves?

Start with 10 calls (1,000 shares of option exposure) and short 520 shares at $S = \$100$. Both lines show **shares per call**: the green call delta changes with stock price, while the blue hedge offset stays fixed.



10 CALLS, SHORT 520 SHARES

Stock	**$100**
Call delta / call	**0.520**
Net shares now	**+0 shares**
Trade to rehedge	**None**

Vertical gap × 1,000 = net shares.

New stock price ($)

80 120

Rebalance through stock and cash

Suppose stock jumps from $\$100$ to $\$110$ while the other model inputs stay fixed.

BEFORE

Call delta ≈ 0.52. Short 520 shares.

AFTER THE MOVE

Call delta ≈ 0.84. The new hedge is short about 842 shares.

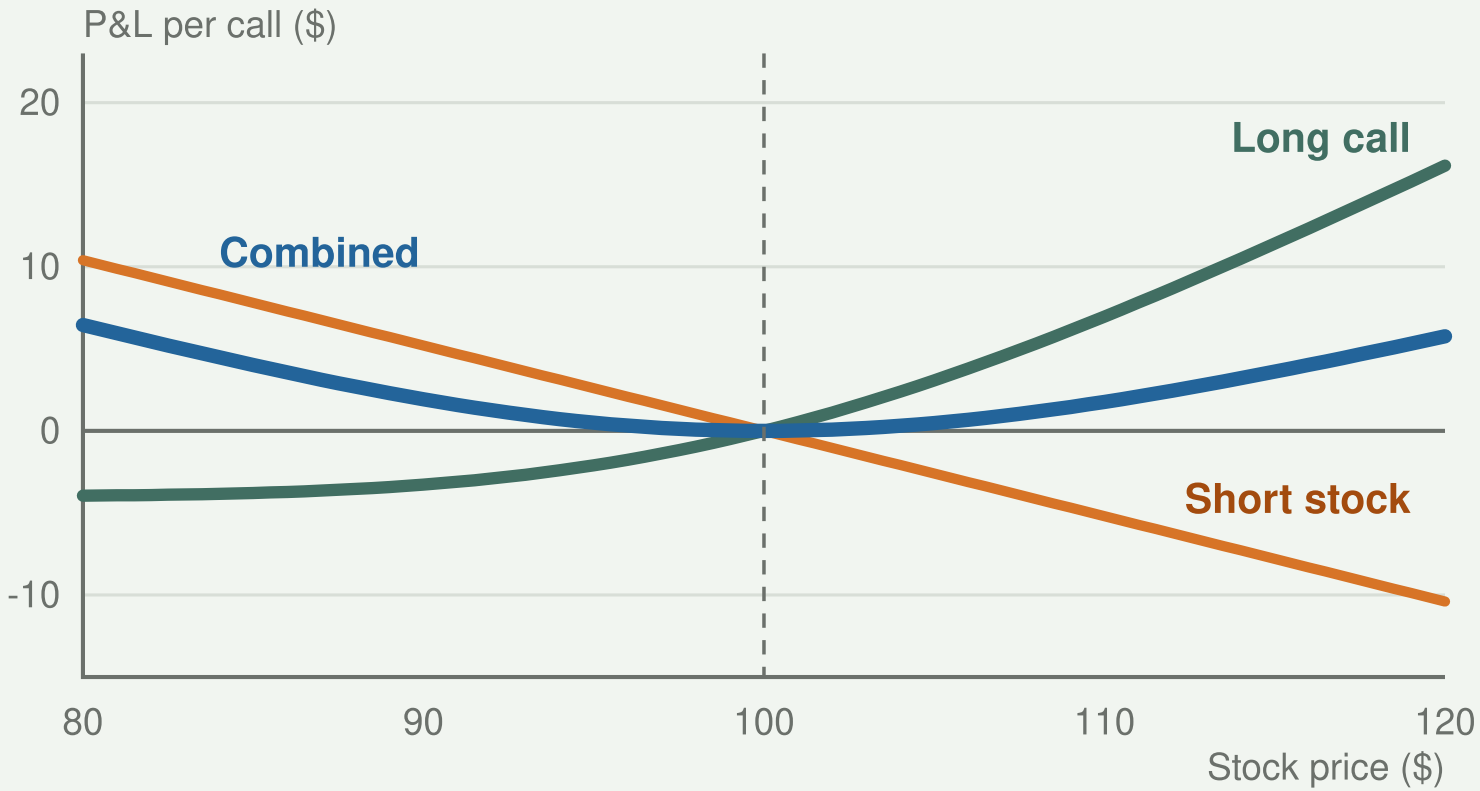
TRADE

Sell about 322 more shares. The sale brings cash into the account.

The hedge is a continuing stock-and-cash trading strategy, not a stock position chosen once at the start.

Delta-neutral is local, not risk-free

The tangent matches the curve at $S = \$100$. It cannot match the whole curve.



A finite move leaves a residual. Lecture 08 will name and measure the curvature behind it.

What has delta hedging not removed?

Time passes. The option value and its hedge change.

Volatility changes. The option price can move with no stock move.

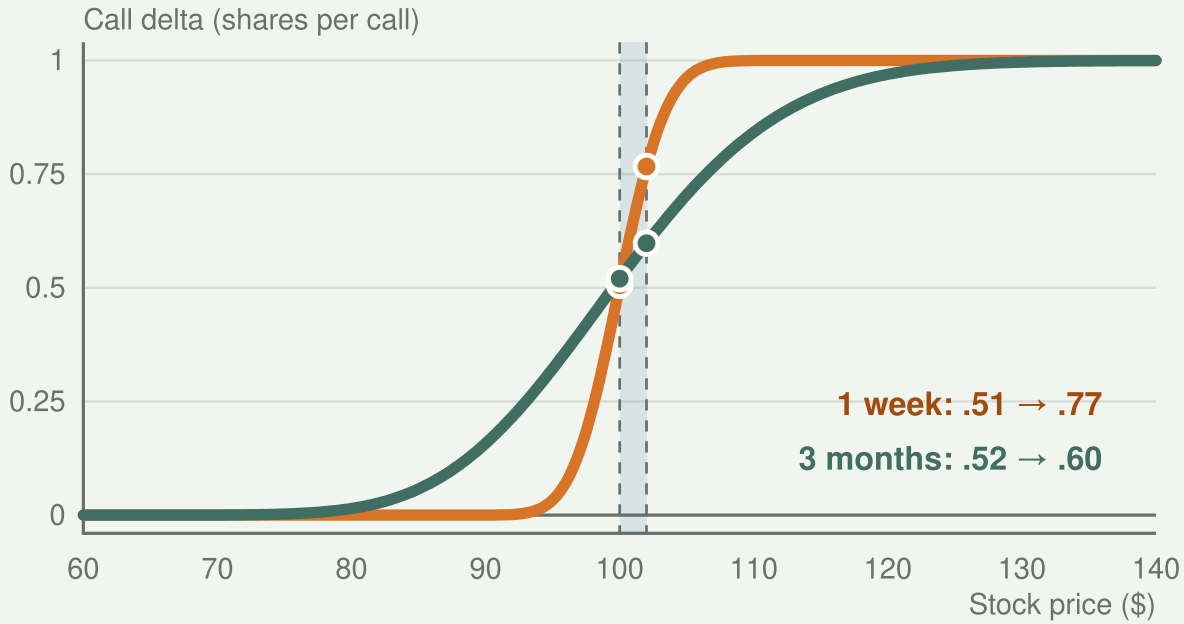
Stock jumps. You cannot rebalance during the jump.

Trading costs money. Frequent hedge updates are not free.

Delta hedging removes one local directional exposure. It does not make an option position riskless.

How quickly does the hedge go stale?

Same strike, same volatility, same $100 starting stock price. What does a $2 move do to each call's delta?



The steeper delta curve makes the hedge go stale faster. Lecture 08 calls this slope **gamma**. Rehedging more often reduces the mismatch, but trading costs money.

Summary

1. Delta is the local change in option value for a $1 change in stock price, holding other inputs fixed.
2. The binomial hedge ratio becomes $\partial C / \partial S$ as stock moves become small.
3. Call delta is positive, put delta is negative, and their difference is one share.
4. A stock trade can neutralize the current delta of one option or an entire portfolio.
5. Delta changes as stock moves, so a directional hedge must be rebalanced.

Next: gamma measures how quickly delta changes.