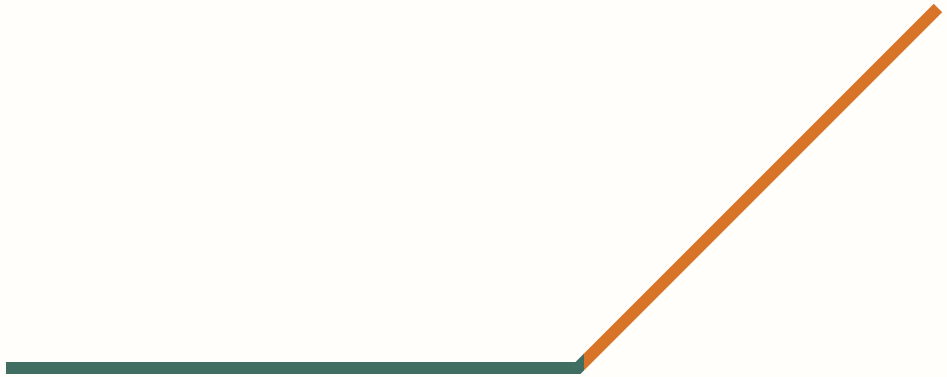
Implied Volatility

Last time: volatility $\longrightarrow$ price.

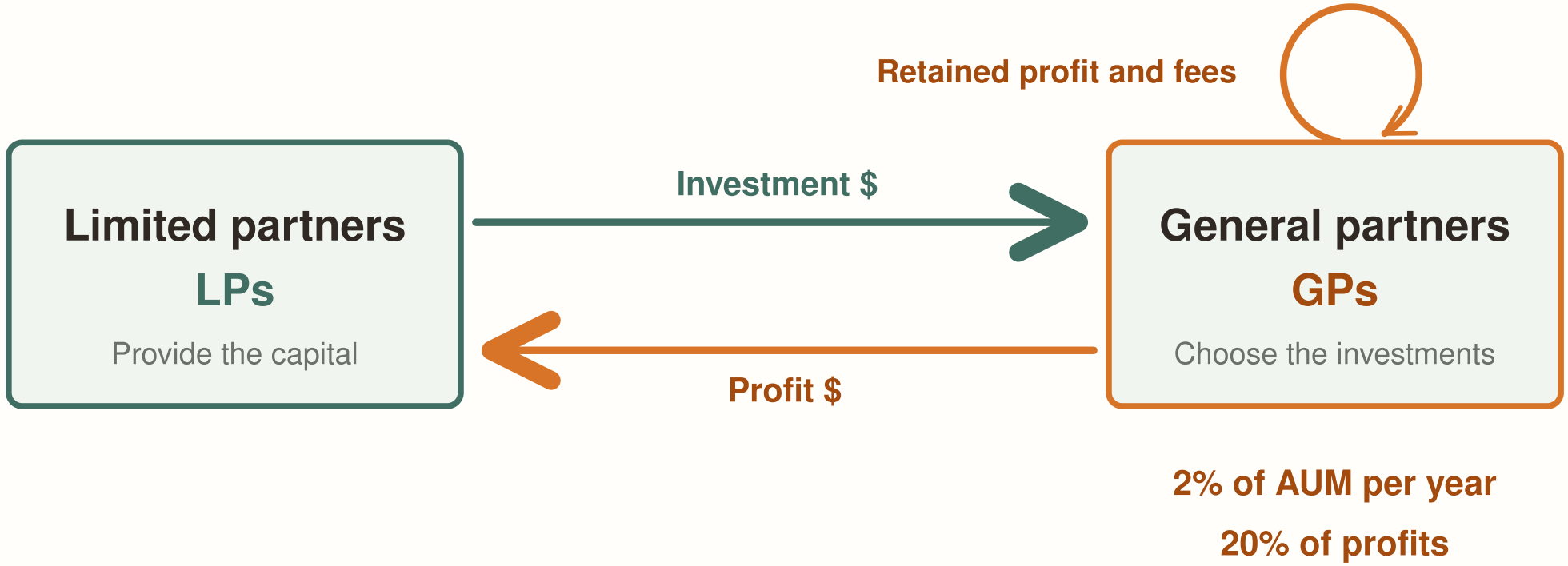
Today: price $\longrightarrow$ volatility.

06



Options in real life

What is a hedge fund?



What are the GPs' incentives?

Same fund, two investments

Safe investment

+10%

Guaranteed gross profit: +$10

Risky investment

+50% or **–50%**

Equally likely. Expected gross profit: $0

	Safe	Risky: up / down (expected)
LP profit	+$6	+$38 / –$52 (–$7)
GP profit	+$4	+$12 / +$2 (+$7)

On expected profit, the LP prefers safe but the GP prefers risky. A performance fee makes the GP **long an option on profits**, so more volatility can appeal even when the investment's expected profit is worse.

Illustration: $100 invested for one year. GP earns 2% of starting AUM plus 20% of positive investment profit. No performance fee on a loss.

Insurance changes incentives

Alpenverein rescue insurance

People who expect to take more mountain risks may be more likely to buy cover. Once insured, a person may also take risks they would otherwise avoid.

Mobile phone insurance

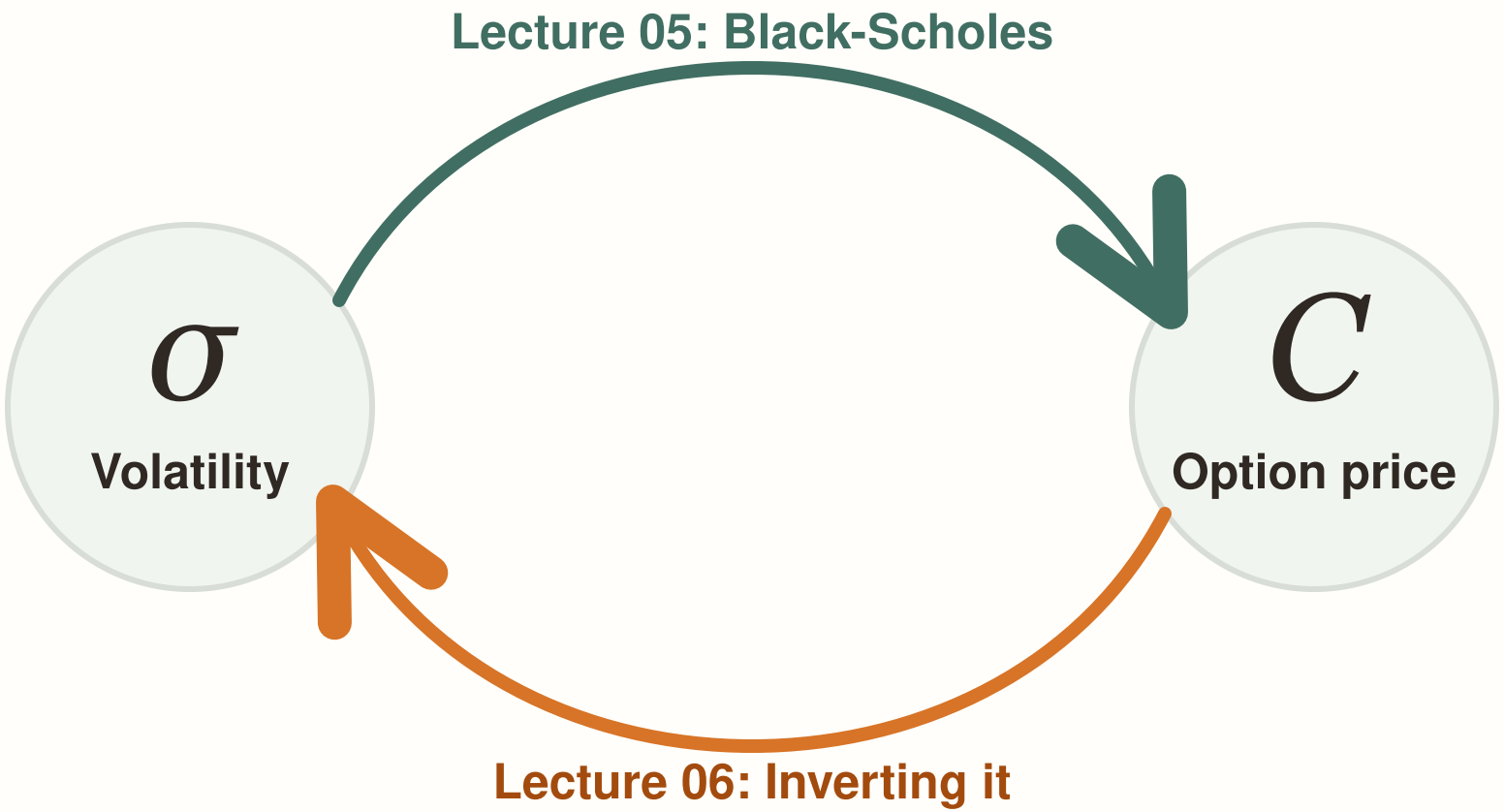
People who expect more damage may be more likely to buy cover. Having cover may also change how carefully they handle the phone.

Two effects to price: Who chooses to buy insurance? How does having it change their behavior?

The common thread: a payoff that depends on the outcome changes the value of uncertainty.

Implied volatility

From volatility to price, and back



A trader's description

> *The wrong number*
> *in the wrong formula*
> *to get the right price.*

Trading options is trading volatility

LECTURE 02

More uncertainty raises the value of calls *and* puts.

LECTURE 03

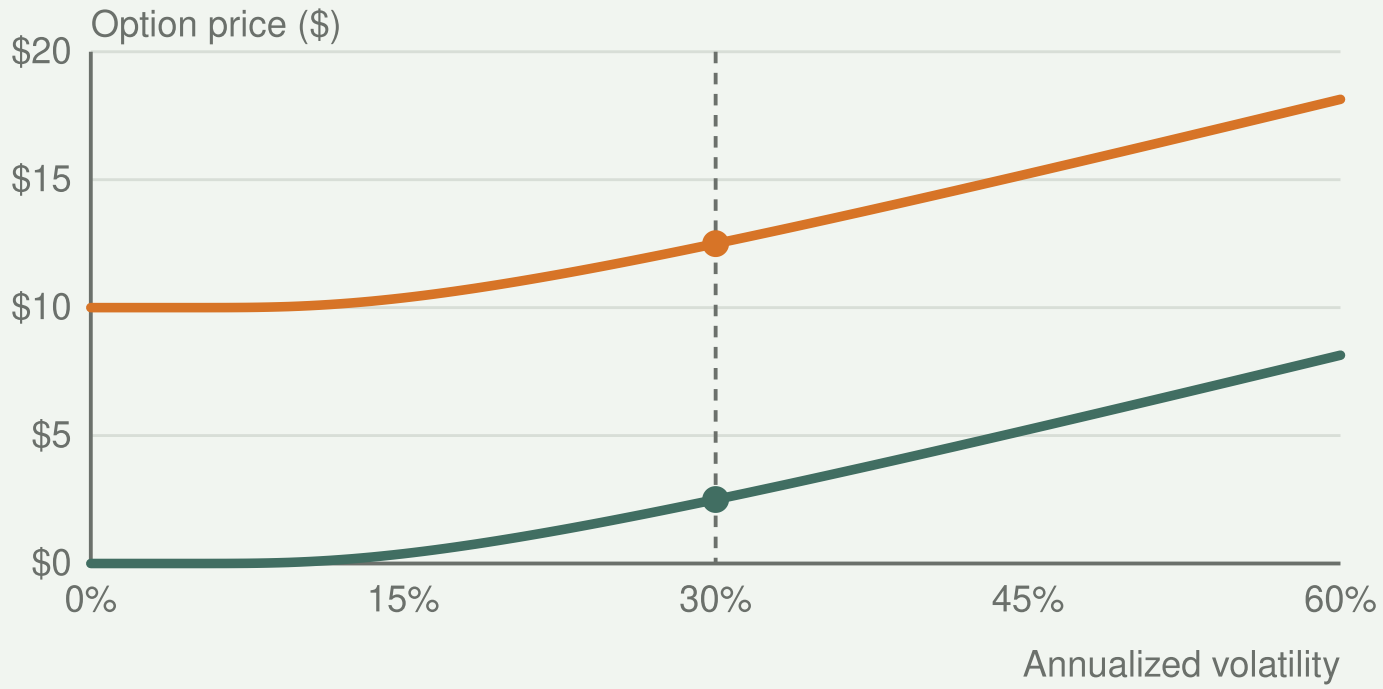
A stock trade transforms calls into puts. Direction and optionality can be separated.

But option premiums depend on too many variables at once to compare directly.

$$C = C_{\mathrm{BS}}(S_0, K, T, r; \sigma)$$

Same strike, same implied volatility

Same stock $S_0 = \$100$, same three-month expiry, same strike $K = \$110$. One call, one put.



Annual volatility 30%

Call · OTM	**$2.50**
Put · ITM	**$12.50**

Put – call = **$10** at every volatility.

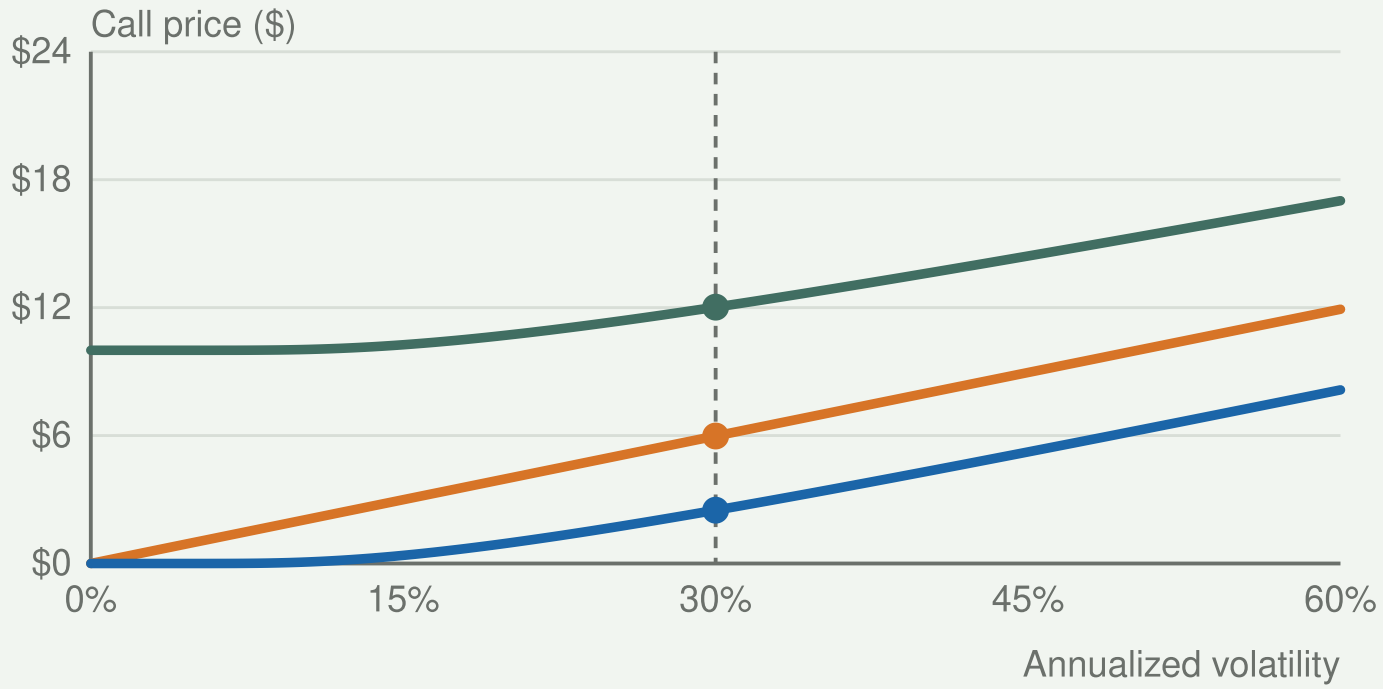
Move the common volatility

At every volatility, $C - P = S_0 - K = -\$10$. If market prices obey parity, the call and put must imply the same volatility.

European options, zero rates and dividends. The graph uses single prices, not bid-ask quotes.

Change only the strike

Same stock $S_0 = \$100$, same three-month expiry. Only the call strike changes.



Annual volatility 30%

K = $90	**$12.02 · 78Δ**
K = $100 · ATM	**$5.98 · 53Δ**
K = $110	**$2.50 · 29Δ**

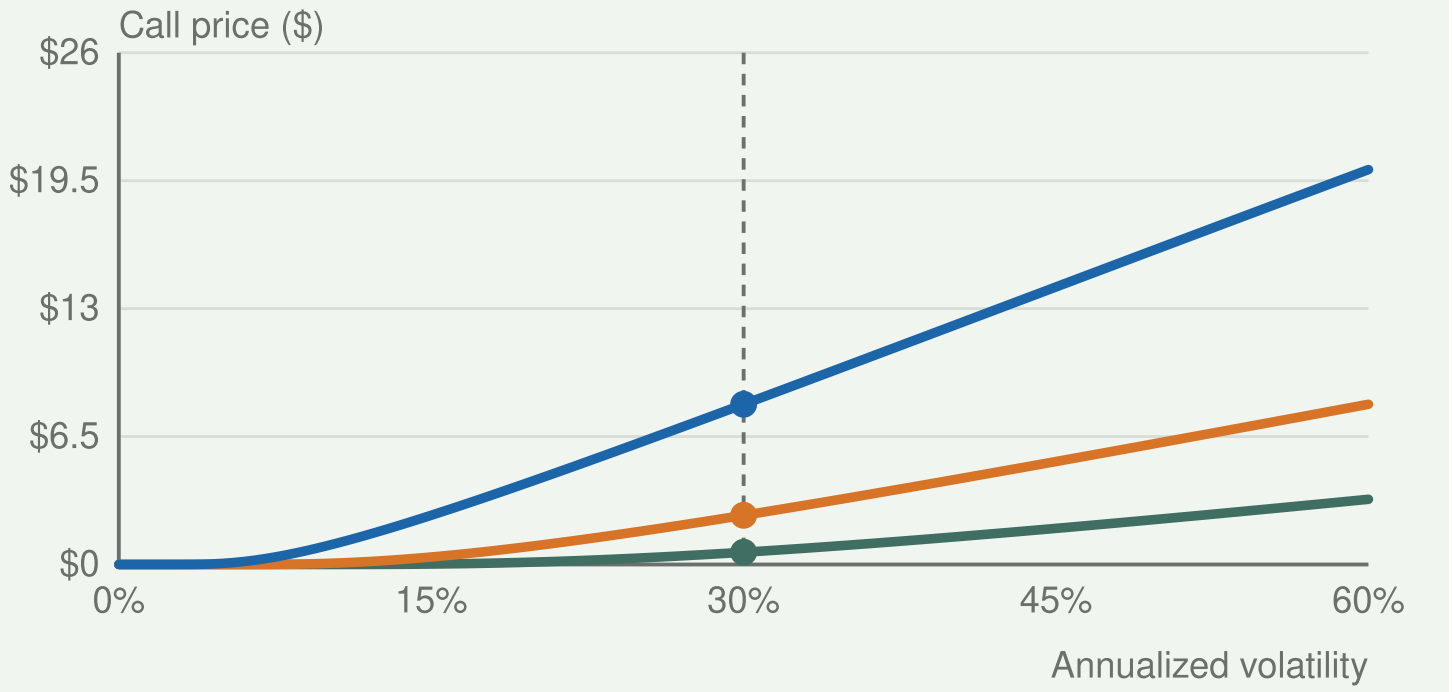
Move the common volatility

Near zero volatility, the $90 and $110 curves flatten. This is zero *vega*, sensitivity to volatility. At 30% vol, their call deltas are about 78 and 29.

All three comparisons: European calls, zero rates, no dividends.

Change only the expiration

Same out-of-the-money call, $S_0 = \$100$ and $K = \$110$. Only the time to expiry changes.



Annual volatility 30%

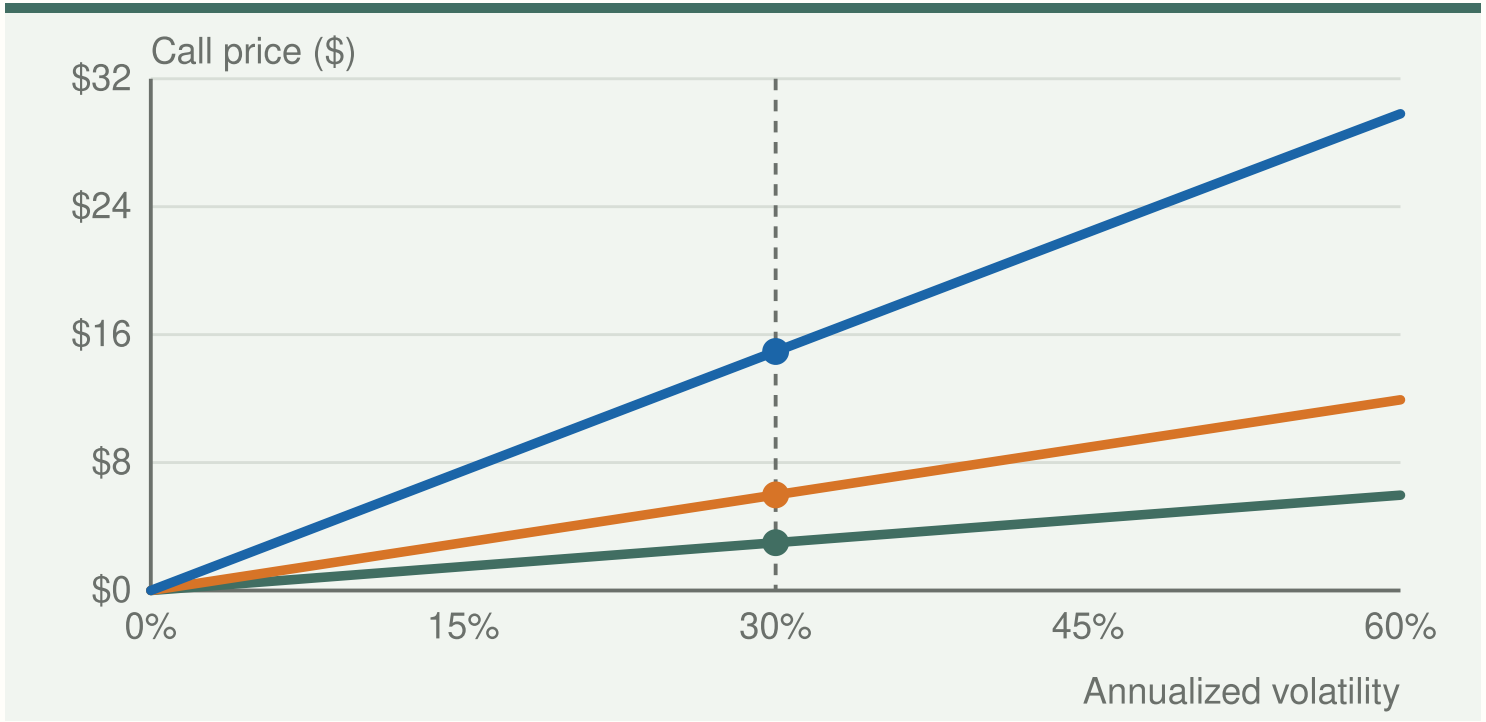
1 month	**$0.62**
3 months	**$2.50**
1 year	**$8.14**

Move the common volatility

At the same volatility, the longer-dated option costs more. The shorter-dated call stays near zero until volatility becomes large enough to make reaching the strike plausible.

Change only the stock price

Each call is at the money with three months to expiry. Stock price and strike scale together.



Annual volatility 30%

$50 stock	**$2.99**
$100 stock	**$5.98**
$250 stock	**$14.95**

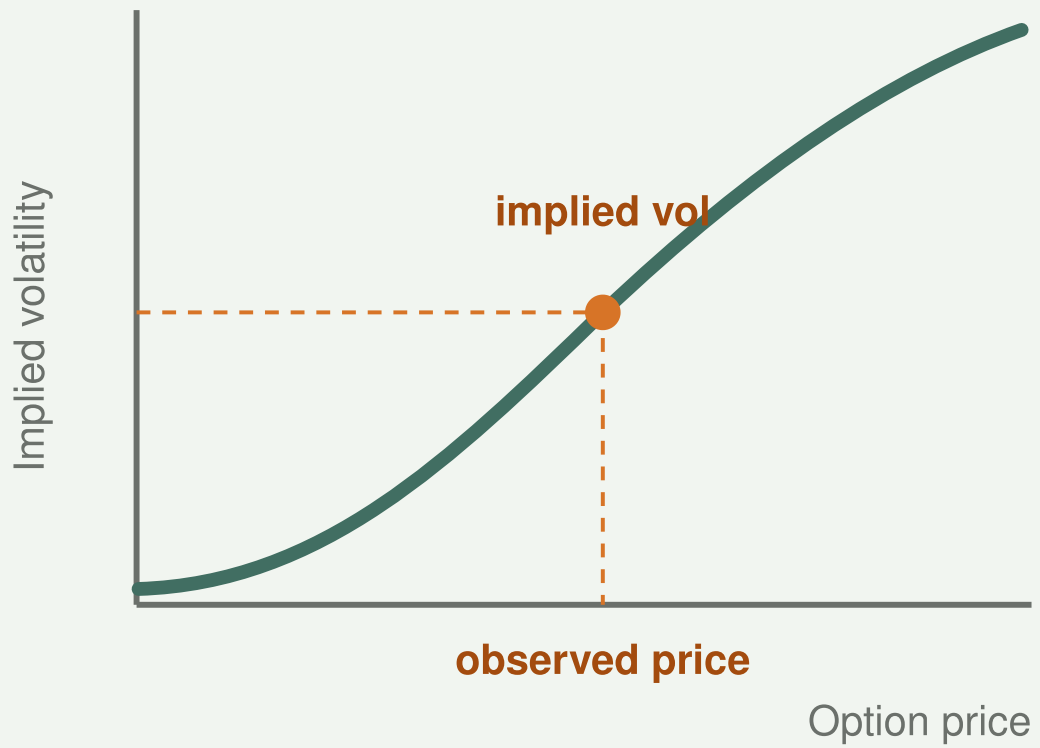
Move the common volatility

Even with identical volatility and moneyness, an option on a $250 stock has a larger dollar premium than one on a $50 stock.

We need a common measuring stick

Volatility is that stick. It lets us compare options with different prices, strikes, expirations, and underlyings.

For a fixed contract, Black-Scholes price rises monotonically with volatility. So we can run the formula backward, from a market price to one implied volatility.



Schematic. Each contract has its own curve. The rising shape is what makes inversion possible.

Read the same markets in volatility space

Illustrative European option boards. Zero rates and dividends.

Option prices ($)

Visa · 1 month Stock $250

CALLS Size	CALLS Bid		CALLS Ask	CALLS Size	STRIKE	PUTS Size	PUTS Bid		PUTS Ask	PUTS Size
(10)	11.75	–	12.25	(10)	240	(10)	1.90	–	2.10	(10)
(10)	5.50	–	6.00	(10)	250	(10)	5.50	–	6.00	(10)
(10)	2.05	–	2.30	(10)	260	(10)	11.75	–	12.50	(10)

Visa · 3 months Stock $250

CALLS Size	CALLS Bid		CALLS Ask	CALLS Size	STRIKE	PUTS Size	PUTS Bid		PUTS Ask	PUTS Size
(10)	14.50	–	16.50	(10)	240	(10)	5.20	–	5.90	(10)
(10)	9.40	–	10.50	(10)	250	(10)	9.40	–	10.50	(10)
(10)	5.60	–	6.30	(10)	260	(10)	15.00	–	17.00	(10)

Mastercard · 1 month Stock $500

CALLS Size	CALLS Bid		CALLS Ask	CALLS Size	STRIKE	PUTS Size	PUTS Bid		PUTS Ask	PUTS Size
(10)	23.75	–	25.00	(10)	480	(10)	4.20	–	4.65	(10)
(10)	11.75	–	12.50	(10)	500	(10)	11.75	–	12.50	(10)
(10)	4.60	–	5.10	(10)	520	(10)	24.00	–	25.50	(10)

The same quotes in implied vol

Visa · 1 month Stock $250

CALLS Size	CALLS Bid		CALLS Ask	CALLS Size	STRIKE	PUTS Size	PUTS Bid		PUTS Ask	PUTS Size
(10)	18.9%	–	21.1%	(10)	240	(10)	19.6%	–	20.5%	(10)
(10)	19.1%	–	20.8%	(10)	250	(10)	19.1%	–	20.8%	(10)
(10)	19.5%	–	20.5%	(10)	260	(10)	18.1%	–	21.4%	(10)

Visa · 3 months Stock $250

CALLS Size	CALLS Bid		CALLS Ask	CALLS Size	STRIKE	PUTS Size	PUTS Bid		PUTS Ask	PUTS Size
(10)	17.6%	–	22.1%	(10)	240	(10)	19.2%	–	20.7%	(10)
(10)	18.9%	–	21.1%	(10)	250	(10)	18.9%	–	21.1%	(10)
(10)	19.3%	–	20.8%	(10)	260	(10)	18.0%	–	22.2%	(10)

Mastercard · 1 month Stock $500

CALLS Size	CALLS Bid		CALLS Ask	CALLS Size	STRIKE	PUTS Size	PUTS Bid		PUTS Ask	PUTS Size
(10)	19.4%	–	22.2%	(10)	480	(10)	20.5%	–	21.5%	(10)
(10)	20.4%	–	21.7%	(10)	500	(10)	20.4%	–	21.7%	(10)
(10)	20.5%	–	21.6%	(10)	520	(10)	19.2%	–	22.4%	(10)

Visa is centered near 20% and Mastercard near 21%. Wider in-the-money spreads and rounded dollar quotes add small differences to the displayed implied vols.

Pause



There’s just one little problem

Black-Scholes gives us the call price when we choose a volatility.

$$C_{\mathrm{BS}}(\sigma) = S_0 N(d_1) - K N(d_2)$$

FORWARD

σ → option price

Plug in a volatility. Calculate the price.

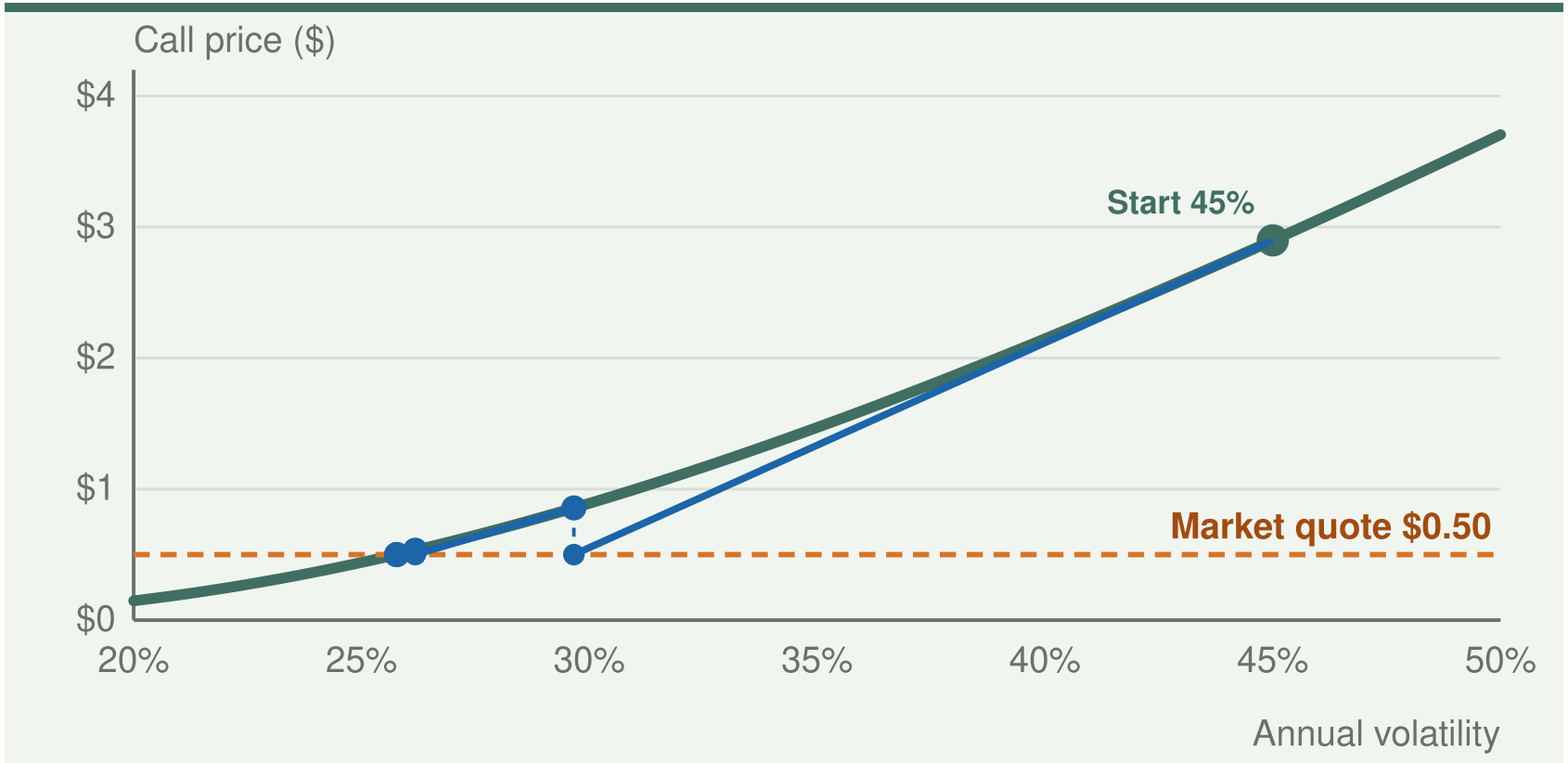
BACKWARD

option price → σ?

There is no similar closed-form formula for this inverse. We have to find the volatility numerically.

Newton's method finds the implied volatility

$S_0 = \$100$, $K = \$120$, $T = 3$ months, $r = 0$. The market call price is $\$0.50$. Start at 45% volatility.



Four tangent steps

1. 45.00% → **29.66%**
2. 29.66% → **26.18%**
3. 26.18% → **25.78%**
4. 25.78% → **25.77%**

Implied volatility
≈ 25.77%

At each guess, follow the curve's tangent to the market-price line. Its crossing gives the next volatility.

Can every price have an implied volatility?

No. Newton's method needs a price that the model can actually reach.

$$\underbrace{\max(S_0 - Ke^{-rT}, 0)}_{\sigma=0} \leq C_{\text{market}} < \underbrace{S_0}_{\sigma\to\infty}$$

Too low: below the zero-volatility price. No model volatility fits.

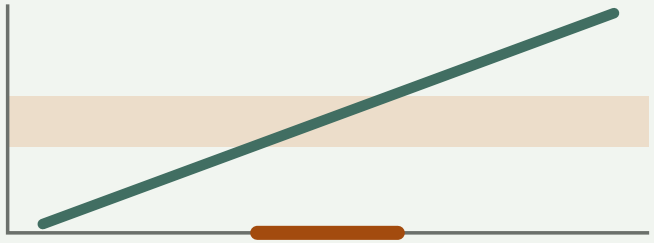
Too high: at or above the stock price. No finite model volatility fits.

European call, no dividends, nonnegative rates. At the lower bound, $\sigma = 0$; at the upper bound, volatility only approaches infinity. Check executable bid and ask prices before calling a quote an arbitrage.

A flat price curve makes the answer fragile

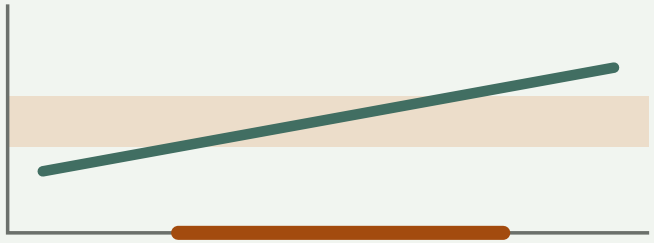
Newton follows a tangent. What if the price curve is almost flat?

Steep price curve



A small quote change gives a small change in implied volatility.

Flat price curve



The same quote change gives a much larger change in implied volatility.

Near a flat part of the curve, a small bid-ask spread or tick-size change can move the fitted volatility a lot. The tangent in Newton’s method is nearly horizontal, so its next guess may jump far away.

Wrong rates make parity look broken

Stock 100. Three months to expiry. The cash rate is 2%, but we convert prices to volatility using 0%.

Executable option prices ($)

CALLS				STRIKE	PUTS			
Size	Bid		Ask	Size	Size	Bid		Ask	Size
(10)	16.24	–	16.44	(10)	85	(10)	0.81	–	1.01	(10)
(10)	12.25	–	12.45	(10)	90	(10)	1.81	–	2.01	(10)
(10)	8.86	–	9.06	(10)	95	(10)	3.38	–	3.58	(10)
(10)	6.12	–	6.32	(10)	100	(10)	5.62	–	5.82	(10)
(10)	4.03	–	4.23	(10)	105	(10)	8.51	–	8.71	(10)
(10)	2.53	–	2.73	(10)	110	(10)	11.99	–	12.19	(10)
(10)	1.51	–	1.71	(10)	115	(10)	15.94	–	16.14	(10)

Implied vol using 2% rates

CALLS				STRIKE	PUTS			
Size	Bid		Ask	Size	Size	Bid		Ask	Size
(10)	29.0%	–	31.0%	(10)	85	(10)	28.9%	–	31.0%	(10)
(10)	29.3%	–	30.7%	(10)	90	(10)	29.3%	–	30.7%	(10)
(10)	29.5%	–	30.6%	(10)	95	(10)	29.4%	–	30.5%	(10)
(10)	29.5%	–	30.5%	(10)	100	(10)	29.5%	–	30.5%	(10)
(10)	29.5%	–	30.5%	(10)	105	(10)	29.5%	–	30.5%	(10)
(10)	29.4%	–	30.6%	(10)	110	(10)	29.4%	–	30.6%	(10)
(10)	29.3%	–	30.7%	(10)	115	(10)	29.3%	–	30.7%	(10)

With the correct 2% rate, call and put vol quotes overlap near 30%. The crossed vol was an input error, not free money.

A large dividend can break a European IV fit

Stock 100. A $30 special dividend arrives tomorrow. These are American options with three months to expiry.

Executable option prices ($)

CALLS				STRIKE	PUTS			
Size	Bid	Ask	Size		Size	Bid	Ask	Size
(10)	80.00	80.10	(10)	20				
(10)	70.00	70.10	(10)	30				
(10)	60.00	60.10	(10)	40	(10)	0.03	0.08	(10)
(10)	40.00	40.10	(10)	60	(10)	2.60	2.70	(10)
(10)	20.00	20.10	(10)	80	(10)	13.44	13.54	(10)
(10)	0.69	0.74	(10)	100	(10)	30.67	30.77	(10)
(10)	0.10	0.15	(10)	120	(10)	50.08	50.18	(10)

American-model interpretation

CALLS				STRIKE	PUTS			
Size	Bid	Ask	Size		Size	Bid	Ask	Size
(10)	ex.	ex.	(10)	20				
(10)	ex.	ex.	(10)	30				
(10)	ex.	ex.	(10)	40	(10)	46.0%	52.3%	(10)
(10)	ex.	ex.	(10)	60	(10)	49.5%	50.5%	(10)
(10)	ex.	ex.	(10)	80	(10)	49.6%	50.4%	(10)
(10)	49.5%	50.3%	(10)	100	(10)	49.2%	50.8%	(10)
(10)	48.4%	51.3%	(10)	120	(10)	47.0%	52.7%	(10)

With American exercise, these calls do not identify a unique IV. The other quotes cluster near 50%.

Implied is not the same as expected

Both are expressed in annualized volatility units, but they answer different questions.

Forecast volatility

How much we expect the stock to move in the future. Compare this forecast with what actually happens later.

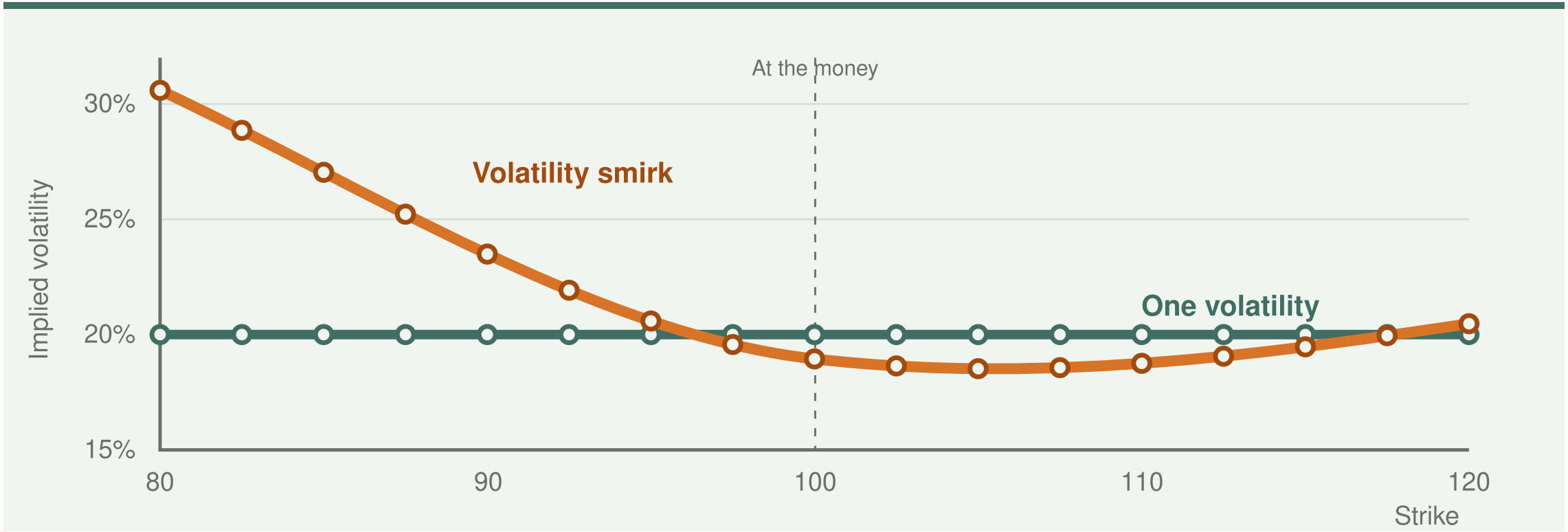
Implied volatility

The input that makes a chosen model reproduce today's option quote. It summarizes the price, not just a prediction of future moves.

These can differ because traders may pay extra for protection, because supply and demand affect quotes, or because the pricing model is imperfect.

Looking forward

Black-Scholes tells us there is one volatility. If the world used that price model, every strike would imply the same volatility.



In fact, market quotes show something very different. The implied volatility changes with strike.

Stylized quotes every $2.50 of strike for one expiry. Lecture 09 will study volatility smiles and skews.

Summary

1. Implied volatility is the model input that reproduces an observed option price.
2. For a valid European vanilla price, the price-versus-volatility relationship lets us find it numerically.
3. Bid and ask premiums become bid and ask implied volatilities. Flat price curves make those quotes fragile.
4. Implied volatility depends on model inputs and exercise rules. It is not a direct forecast of future volatility.
5. Different options can imply different Black-Scholes volatilities.

Next: how does an option’s stock exposure change as the stock moves?