What we know so far

	Underlying	Call / Put	Strike	t_{exp} or σ	Style	
ARBITRAGE CONCEPT #1 Strike monotonicity	📌	📌	↔	📌	📌	$P_c(K_L) \geq P_c(K_H)$ $P_p(K_L) \leq P_p(K_H)$ if $K_L < K_H$
ARBITRAGE CONCEPT #2 Time/vol monotonicity	📌	📌	📌	↔	📌	$P(t_1) \leq P(t_2)$ if $t_1 < t_2$ $P(\sigma_1) \leq P(\sigma_2)$ if $\sigma_1 < \sigma_2$
ARBITRAGE CONCEPT #2b American-exercise dominance	📌	📌	📌	📌	↔	$P_A \geq P_E$
ARBITRAGE CONCEPT #3 Put-call parity	📌	↔	📌	📌	📌	**Today's topic**

Moving money through time

Suppose we invest **$100 today** for **one year** at **10% per year**.

Simple interest
Interest added once

$100
today

× (1 + 0.10)

$110
in one year

Periodic interest
Twice a year

$100
today

× 1.05

$105
six months

× 1.05

$110.25
one year

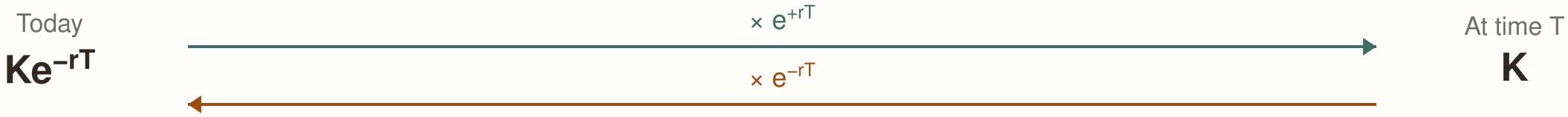
Continuous interest
Infinitely often

$100
today

× $e^{0.10}$

$110.52
in one year

As the number of interest payments grows, $(1 + r/n)^{nT} \longrightarrow e^{rT}$.



The principle of replication

> *"If two different things result in the same future outcome, they must be worth the same now."*

The share

Buy it directly today.

Options and cash

Can they *replicate* holding stock at expiry?

Matching outcomes let us (a) equate prices, (b) identify an arbitrage, and (c) build a hedge.

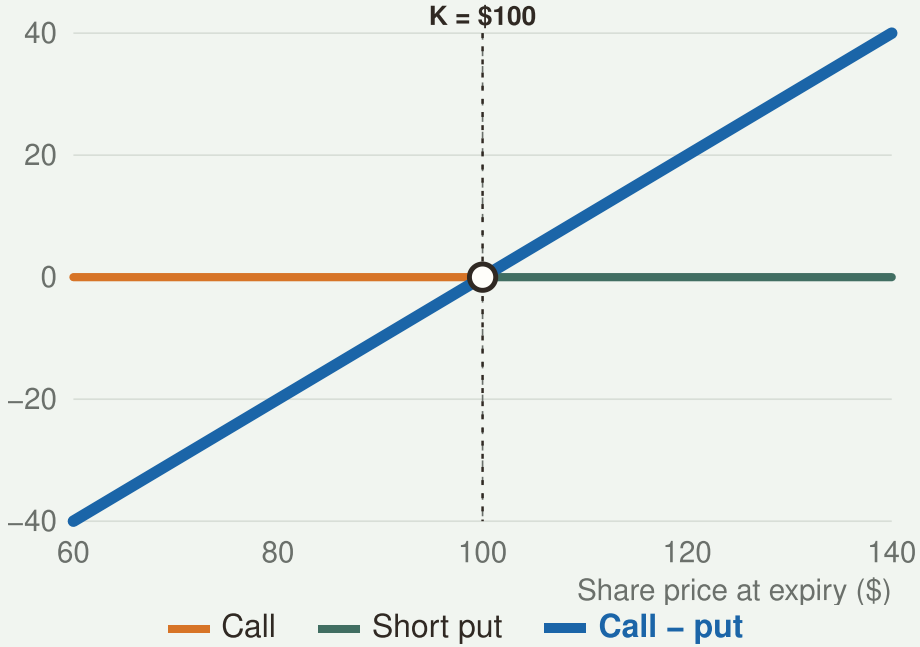
A call plus a short put

Same share, same strike $K = \$100$, same expiry.

Long call: the right to buy the share for $100.

Short put: the obligation to buy it for $100 if assigned.

Move the expiry price to see how the two payoffs meet at the strike.



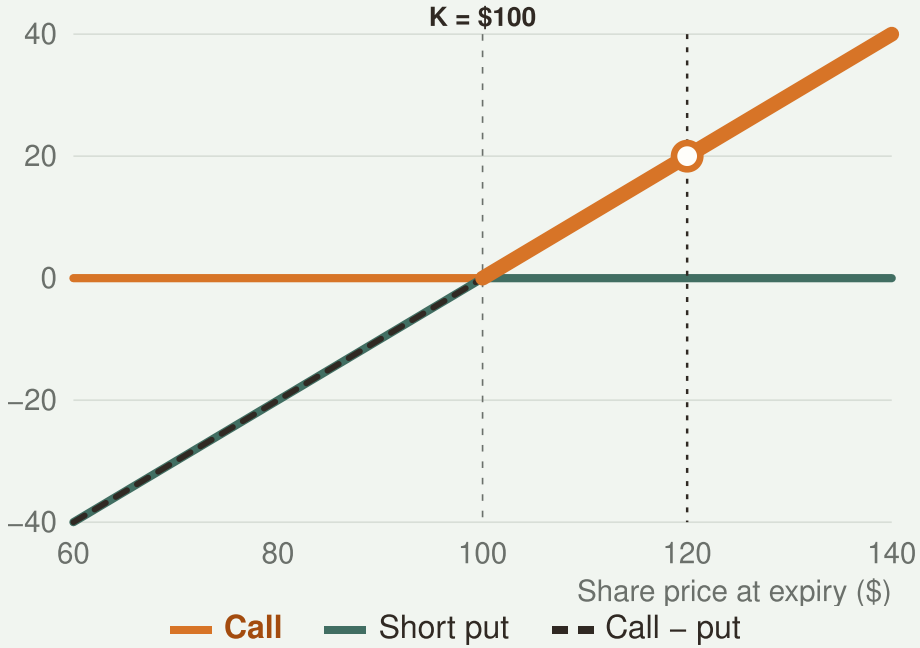
Below K, the short put carries the position. Above K, the long call does. Their sum is one straight line.

Above the strike: exercise the call

At $S_T = \$120$, the call is worth **$20**. Exercise it: receive one share and pay $100.

The short put expires with no value.

Combined payoff: +$20. You own one share and have paid $100.

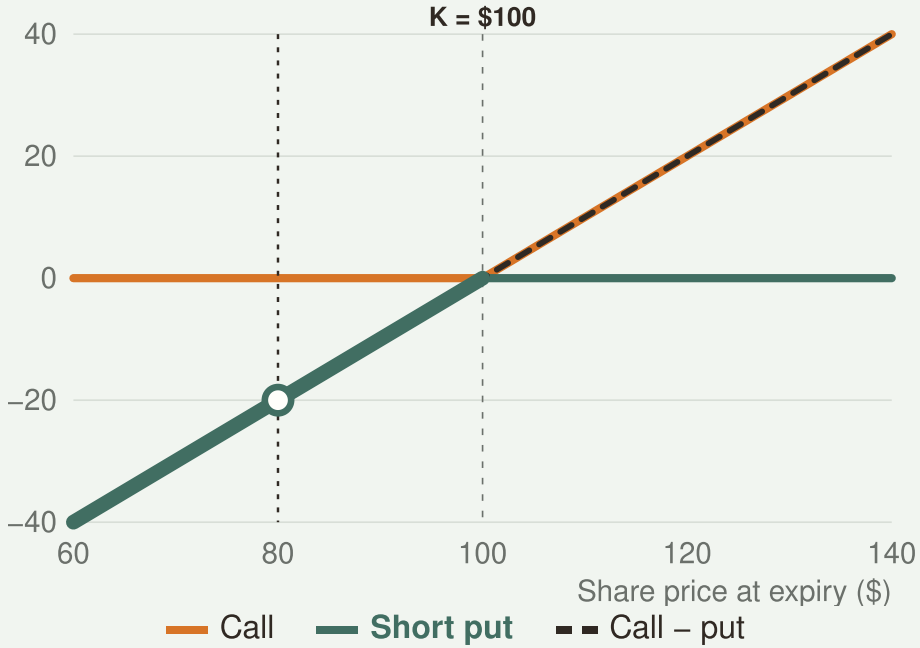


Below the strike: the put is assigned

At $S_T = \$80$, the call expires with no value.

The short put is worth **–$20** to you. The holder exercises: you receive one share and pay $100.

Combined payoff: –$20. You still own one share and have paid $100.



At $80: call $0 + short put –$20 = –$20

The two options make a forward

At time T:

$$C_T - P_T = (S_T - K)^+ - (K - S_T)^+$$
$$= (S_T - K)^+ + (S_T - K)^- = S_T - K$$

Here $x^- = \min(x, 0)$ is the signed negative part.

The share is physical. Think of it as the piece of paper representing one share: you end up long one share.

The strike is future cash. You pay K at expiry, whether through call exercise or put assignment.

Long call minus long put = long share delivered at T, with K due at T.

What is that worth today?

The share is worth S_0 today. The K payment is due later, so set aside only its present value now.

$$\mathrm{PV}(K) = Ke^{-rT}$$

Investing Ke^{-rT} at a continuously compounded risk-free rate r grows to K at T.

Assume European options, no dividends and one common funding rate for now.

The same payoff, priced today

Both portfolios have payoff $S_T - K$ at expiry. What does each one cost **at time 0**?

Call minus put

Buy the call for C_0. Sell the put and receive P_0.

Net cost today: $C_0 - P_0$.

Share minus future debt

Buy one share for S_0. Borrow Ke^{-rT} now and repay K at expiry.

Net cost today: $S_0 - Ke^{-rT}$.

Same payoff at T means the two costs at time 0 must be equal.

Put-call parity

ARBITRAGE CONCEPT #3

$$C_0 - P_0 = S_0 - Ke^{-rT}$$

$$e^{-rT} \approx 1 - rT$$

For small rT, this gives $C_0 - P_0 \approx S_0 - K + KrT$.

The KrT term is just the cost of borrowing K for time T at rate r.

Find the arbitrage

- Stock price: $S_0 = \$110$.
- Matching call and put: strike $K = \$100$, same expiry.
- Call minus put: **$9**.

What would you do?

The synthetic share is $1 too cheap.

Fair value: $C_0 - P_0 = S_0 - K = \$10$.

Buy the call, sell the put, short one share, and set aside $100 for the strike.

Assume zero rates and dividends.

Lock in the dollar

Today

Short stock: **+$110**

Buy call, sell put: **–$9**

Set aside strike cash: **–$100**

Net: +$1

At expiry

The options deliver one share for $100. Use the reserved cash to pay the strike, then return the share to close the stock short.

Net: $0, whatever S_T

If the option portfolio were too expensive, reverse all four positions.

Idealized trade: executable prices, no stock-borrow fee, no margins, and no transaction costs.

Options board: pick'em markets

Stock price

$53.10 (pick'em market)

Fixed carry across strikes

$K(1 - e^{-rT}) \approx KrT = \$0.30$

Put-call parity

$C - P = S - K + KrT$

CALLS					STRIKE	PUTS				
Bid size	Bid		Ask	Ask size		Bid size	Bid		Ask	Ask size
(10)	18.40	—	18.40	(10)	35	(10)	0.00	—	No offer	
(10)	13.35	—	13.35	(10)	40	(10)	0.00	—	No offer	
(10)	8.50	—	8.50	(10)	45	(10)	0.10	—	0.10	(10)
(10)	3.95	—	3.95	(10)	50	(10)	0.55	—	0.55	(10)
(10)	1.00	—	1.00	(10)	55	(10)	2.45	—	2.45	(10)
(10)	0.25	—	0.25	(10)	60	(10)	6.85	—	6.85	(10)
(10)	0.00	—	No offer		65	(10)	11.60	—	11.60	(10)
(10)	0.00	—	No offer		70	(10)	16.50	—	16.50	(10)

Can you find all the free money?

Pause



Dividends

A dividend is cash leaving the company. All else equal, the share is worth about D less once it goes ex-dividend, including at expiry.

Share before payout **– *D* cash** → Ex-dividend share

Own the share

Buy it for S_0 today. Collect the dividend before expiry, then still own the share at T.

Use the options

The call-minus-put delivers the share at T for K, but does **not** collect the intervening dividend.

For a known dividend worth D_0 today: $C_0 - P_0 = S_0 - D_0 - Ke^{-rT}$.

When to exercise an American call early

Just before the ex-dividend date, an in-the-money call holder has a choice.

Exercise now

Pay K now, own the share, and receive the dividend D. Give up the call's remaining optionality.

Keep the call

Keep the option's time value and postpone paying K, but miss the dividend.

Exercise just before ex-dividend when **dividend D exceeds the remaining optionality plus the benefit of delaying K.**

With negligible interest, the rule is simply D greater than the remaining time value. If you can sell the call above intrinsic value, sell it and buy the share instead of exercising.

Borrow and lend cash

For European options, set dividends aside for a moment. In reality there are different rates for cash you borrow and cash you lend.

r_{short} = rate paid to borrow cash. r_{long} = rate earned when lending cash. Usually $r_{\text{short}} > r_{\text{long}}$.

Borrow cash: r_{short}

To replicate $C - P$, buy the share and borrow the cash that will become K at expiry.

$$C - P \leq S_0 - Ke^{-r_{\text{short}}T}$$

Lend cash: r_{long}

To replicate $P - C$, short the share and invest cash that will become K at expiry.

$$P - C \leq Ke^{-r_{\text{long}}T} - S_0$$

$$S_0 - Ke^{-r_{\text{long}}T} \leq C - P \leq S_0 - Ke^{-r_{\text{short}}T}$$

This gap is the **funding spread**. In this class, we ignore it for simplicity.

When to exercise an American put early

A deep in-the-money put can be exercised before expiry. The attraction is receiving K **now**, not later.

Exercise now

Sell the share for K today. Invest the proceeds and earn interest until expiry.

Keep the put

Keep the choice to sell later. If the share rebounds, you need not exercise.

Exercise when the interest gained by receiving K early exceeds the value of keeping that choice.

Higher rates and less remaining optionality favor exercise. A coming dividend favors waiting, since the ex-dividend share-price drop helps the put. If the put can be sold above its exercise value, sell it instead.

Stock borrow can erase the arbitrage

Return to our $S_0 = \$110$, $K = \$100$, $C - P = \$9$ trade. Its clean parity value is $\$10$, so the apparent edge is $\$1$.

Borrow the share

The trade shorts one share for six months. At a fixed **2% annual borrow fee** on $\$110$:

$$B_0 \approx 110 \times 0.02 \times 0.5 = \$1.10$$

Adjust the parity test

The cheap-synthetic side has a lower bound, not the clean equality:

$$C - P \geq S_0 - Ke^{-rT} - B_0$$

Here, with $r = 0$, that lower bound is $\$8.90$. The quoted $\$9$ is not an arbitrage.

The $\$1$ clean edge is smaller than the $\$1.10$ borrow bill. A changing fee or recall makes the short even less certain.

Options board: bid–ask markets

Stock price

$53.10 (pick'em market)

Fixed carry across strikes

$K(1 - e^{-rT}) \approx KrT = \$0.30$

Put-call parity

$C - P = S - K + KrT$

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Can you find all the free money?

A call is a put plus a stock trade

$$C_0 = P_0 + S_0 - Ke^{-rT}$$

Long put

Own the right to sell at K.

Long share

Add the share's directional exposure.

Borrow strike cash

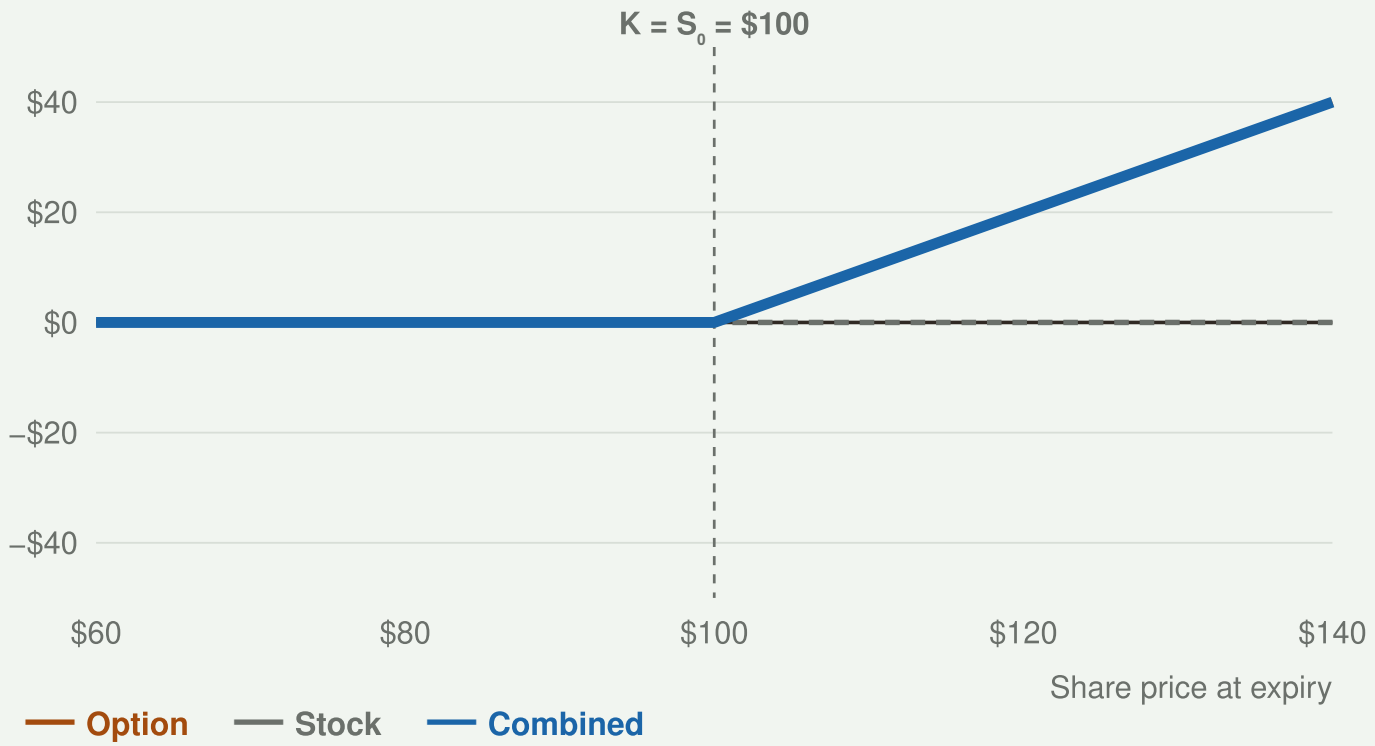
Borrow Ke^{-rT} now. Repay K at expiry.

Strip away stock and financing. A call and put contain the same optionality.

Matching European options. Idealized funding and stock trades, with no dividends or trading costs.

You can transform calls and puts into each other

Choose an option, then move the stock hedge. The stock changes the slopes, but not the kink.



OPTION POSITION

Stock position **0 shares**

No stock hedge. The combined line is the option alone.

$K = S_0 = \$100$. Expiry values per share, before option premium, funding and trading costs.

Trade the view you actually have

My view is mainly directional

“I think the share will rise.”

The share or a forward expresses that view more directly.

My view is about the distribution

“I think the market is underpricing large moves.”

An option buys exposure to the tails. A stock hedge can remove much of its direction.

The width and shape of returns matter, not just their mean.

An unhedged call or put still has direction. The distinct thing you buy or sell is optionality.

Next: ok but what are these options *worth*?

- So far, we have learned to relate the prices of options to each other.
- But we still do not know how to price an option on its own.
- Given a model for the share-price distribution at expiry, how do we calculate the option's value?

We will start answering that question in Lecture 04. Finally.

Summary

1. Long call plus short put makes you buy the share for K at expiry.
2. Under clean European assumptions, $C_0 - P_0 = S_0 - Ke^{-rT}$.
3. Rearranging parity shows a call is a put plus a financed share trade.
4. Long calls and puts both own optionality. Short calls and puts both sell it.
5. If you have a directional opinion, trade the stock.
6. Real trades still face dividends, funding, borrow, execution and exercise frictions.

Calls and puts differ in direction, not in the kind of optionality they contain.