What we learned in Lecture 01

1. A **call** gives its holder the right to buy at the strike without requiring exercise.
2. A **put** gives its holder the right to sell at the strike without requiring exercise.
3. At expiry, the holder exercises only when doing so has positive value.
4. The holder pays a premium; the short option can be assigned and must perform.
5. Option-like rights appear in finance, insurance, contracts and everyday life.

We defined the right. Now we need to value it.

What is a right without an obligation worth?

You can take the good outcomes.

You can walk away from the bad ones.

The right

Keep the upside if it happens.

No obligation

Reject the downside if it happens.

Key question for today:

What determines the value of that asymmetry?

Meet Boring Utility Co.

Share price today

$100

A regulated utility with predictable cash flows.

- No major announcements expected
- Few operational surprises
- Three months from now, it will probably be worth about **$100**.

Very little uncertainty about the outcome.

Now meet Binary Biotech

Share price today: **$100** · Drug-trial result arrives in three months.
Possible outcomes:

Drug fails	**$50**
Drug works	**$400**

What probabilities of success and failure are implied by today's $100 price?

$$100 = q(400) + (1 - q)(50) \quad \Rightarrow \quad q = \frac{1}{7} = 14.3\%$$

Success: **14.3%** · Failure: **85.7%**

Simplified risk-neutral probabilities. Assume zero rates, no dividends and exactly two expiry outcomes.

Same call. Very different value.

Three-month call · strike $K = \$100$

Boring Utility Co.

Almost surely $S_T = \$100$:

$$C_0 \approx \max(100 - 100, 0) = \$0$$

No upside surprise to capture.

Binary Biotech

Using the implied probabilities:

$$C_0 = \frac{1}{7}(400 - 100) + \frac{6}{7}(0)$$

$$C_0 = \$42.86$$

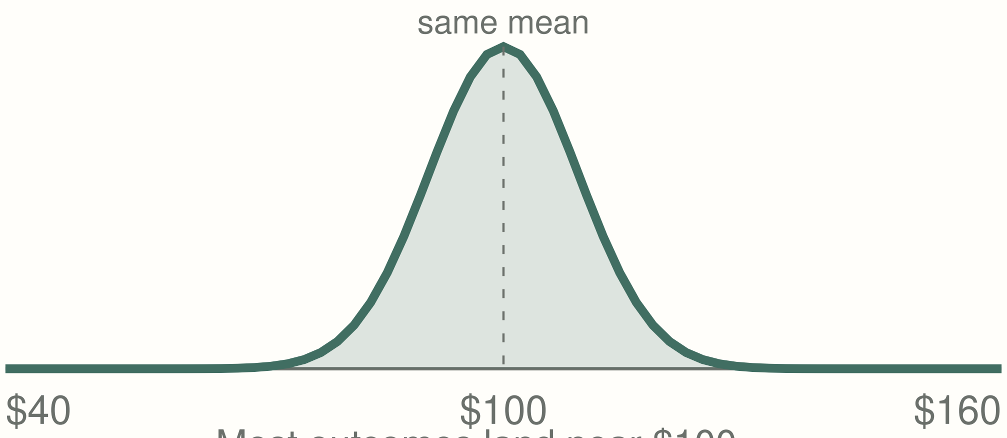
Same spot. Same strike. Same average future stock price. **Different uncertainty.**

Undiscounted toy model. Zero rates and dividends.

Uncertainty creates option value

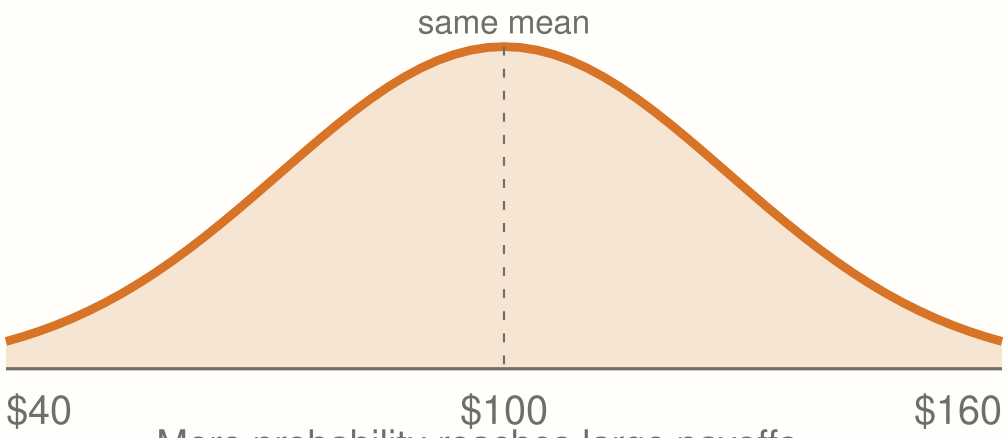
Options are worth more when outcomes are more uncertain.

Tight distribution



Most outcomes land near $100.

Wide distribution



More probability reaches large payoffs.

This naturally leads us from thinking about outcomes to thinking about **distributions**.

Options are bets on distributions

Options **are** bets on volatility. More generally, they are bets on the shape of probability distributions.

- A stock payoff is linear.
- A call payoff has a **kink**: losses stop at zero while gains keep growing.

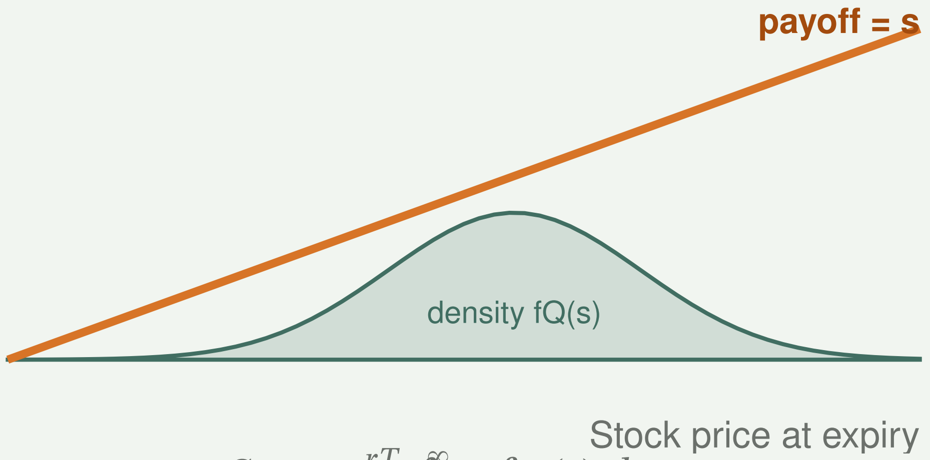
$$C_T = (S_T - K)^+$$

Convex payoff

This non-linearity makes the *distribution* of outcomes matter, not only their average.

Expected payoff is an integral

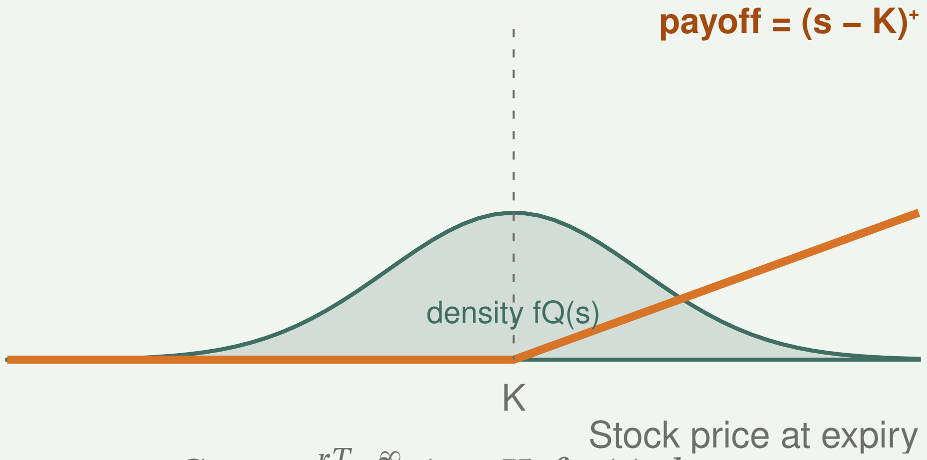
Stock · linear payoff



$$S_0 = e^{-rT} \int_0^\infty s\, f_Q(s)\, ds$$

Only the mean matters.

Call · non-linear payoff



$$C_0 = e^{-rT} \int_K^\infty (s - K) f_Q(s)\, ds$$

The shape of the distribution matters.

Why is early exercise usually suboptimal?

An unexpired option contains two things:

Intrinsic value

What exercise gives you **today**.

+

Time value

The remaining chance that uncertainty creates something better.

Exercise keeps the intrinsic value but destroys the remaining time value.

If you want to exit, selling the option is usually better than exercising it.

Important exceptions and complications: dividends, stock borrow, interest rates, puts, transaction costs and contract details.

More time cannot hurt an American option

ARBITRAGE CONCEPT #2

Two otherwise identical American options expire at T_1 and T_2, with $T_2 > T_1$.

Shorter expiry · T_1

Exercise at any time through T_1.

$\subset$

Longer expiry · T_2

Every earlier exercise date, plus more.

$$V_A(T_2) \geq V_A(T_1)$$

The longer-dated American option contains all the rights of the shorter-dated option.

European options: more time usually adds value, but this is not pure dominance. The later contract cannot be exercised at T_1, so dividends, rates and carry can affect the comparison.

Summary

1. An option is valuable because it preserves the **good outcomes** without requiring the bad ones.
2. With the same average outcome, a **wider distribution** can make an option worth more.
3. A non-linear payoff depends on the **shape of the distribution**, not only its mean.
4. Exercise captures intrinsic value but gives up the option’s remaining **time value**.
5. More exercise opportunities cannot reduce value, so a longer-dated **American option** cannot be worth less.

Uncertainty creates value when you have the right to choose.