

Welcome!

Agenda for today:

1. Administrative logistics
2. What is an option?
3. What kinds of options are there?
4. What are the parameters that define an option?

01

Learning Goals

At the end of this course, you will be able to:

- Explain what financial options are and how they're used.
- Relate options to each other and find value bounds using arbitrage arguments.
- Price vanilla options using both closed-form and numerical techniques.
- Manage the risks of a portfolio of options.
- Produce and use volatility models of options curves and surfaces.

About me and the course

- I am an electrical engineer by training.
- I began as a trader at Jane Street in 2008.
- By 2011 I was the head of the options desk in London, running all EMEA options trading.
- The content of this course is nearly identical to the intern and new-hire trainings that I conducted for years.

You will learn the same stuff.

Why should you care?

- **Options are everywhere.** Options and option-like payoffs appear throughout finance and everyday life, as you'll see later today.
- **The math is interesting.** Options connect probability, calculus, optimization, and economics.
- **Your compensation may include them.** You may need to value the stock options you receive at the high-flying startup you join or start someday.
- **They can become a career.** You may even want to get into professional trading.

Lecture Sequence

01 What Is an Option?

Contracts, payoffs, and exercise rights

02 The Value of Optionality

Uncertainty, distributions, and time value

03 Put-Call Parity and Synthetic Equivalence

Synthetic positions and static arbitrage

04 Implied Volatility

Black-Scholes and implied-vol quoting

05 Volatility Curves and Surfaces

Smiles, smirks, skew, and surfaces

06 No-Arbitrage Volatility Geometry

Bounds, convexity, and surface arbitrage

07 Market-Implied Distributions

Extracting risk-neutral CDFs and PDFs

08 Financial Time and Variance-Time

Trading time, calendar time, and events

09 Delta and Directional Hedging

Hedge ratios and dynamic hedging

10 Gamma, Theta, Vega, and P&L

Convexity, decay, and volatility P&L

11 The Binomial Pricing Model

Replication, trees, and American exercise

12 Stochastic Volatility Models

Random volatility, jumps, and calibration

Evaluation Touchpoints

Assignments follow the lecture that supplies their core tools.

After Lecture 03

Assignment 1

Put-call parity

After Lecture 06

Assignment 2

Arbitrage-free volatility surface

After Lecture 10

Assignment 3

Hedging an options portfolio

After Lecture 11

Computer Assignment 1

Write a fast options pricer

After Lecture 12

Final Exam

Cumulative course assessment

You can use LLMs and one prompt to solve the assignments. It's up to you whether to do this or not. I would suggest not.

Expectations and schedule

- It is expected that students will have a basic understanding of single-variable calculus and basic programming skills in Python.
- It is *not* expected that students will have prior experience with financial markets or trading.
- We will meet once a week for lectures, and once again for supervisions.

Non-goals

We will not teach you profitable trading strategies.

- Making money in financial markets is extraordinarily difficult.
- Course sellers, TikTok influencers, Zlatan Ibrahimovic, none of them know how.
- They make money off of you, not from their (supposed) trading.



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- Since this is a serious course, no more talk about “how to make money trading”.
 - If this disappoints you then this isn’t the course for you.
 - If you want to learn the basics of financial markets the way pros do, then it is.

Agreed?

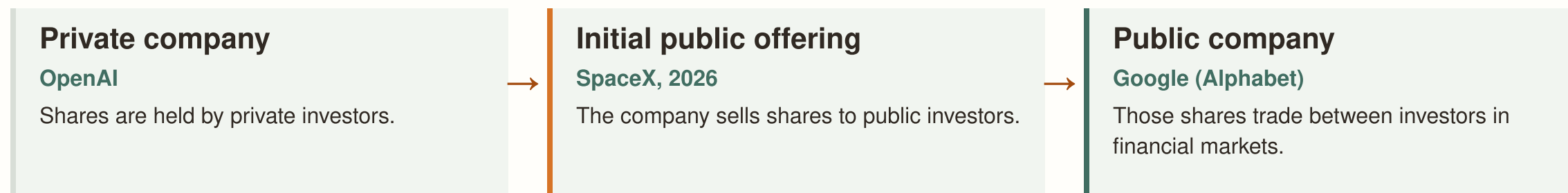


Now let's define an option.

But first: what's a stock?

Assets	=	Liabilities	+	Equity
What the company owns		What the company owes		What belongs to shareholders

Equity is the residual claim: what remains for shareholders after liabilities are paid.



A **stock** is a tradable unit of equity ownership in a company.

Illustrative order book		BID SIZE	PRICE	ASK SIZE
Best market: 100.05 bid / 100.06 ask			100.08	860
			100.07	425
			100.06	160
		125	100.05	
		380	100.04	
		910	100.03	

An option is the right but not the obligation to {buy|sell}.

A **call** gives its holder the right to **buy** the underlying at a fixed price.

A **put** gives its holder the right to **sell** the underlying at a fixed price.

- Note that we said a **right**.
- The holder chooses whether to exercise the right **or not**.
- These are bilateral contracts between two people.

Parameters of an option

Lots of words in the definition.

What do they mean?

Underlying

One share of XYZ

Strike price, K

€100 per share

Expiry, T

Three months from today

Exercise style

European: at expiry only

American: any time before expiry

Settlement type (what changes hands)

Physical: the actual underlying

Cash: A cash amount equivalent to the value

At expiry, take the better of two choices.

$$C_T = \max(S_T - K, 0)$$

Exercise

Buy at K , receive a share worth S_T .

Value: $S_T - K$

Let it expire

If buying is unattractive, do nothing.

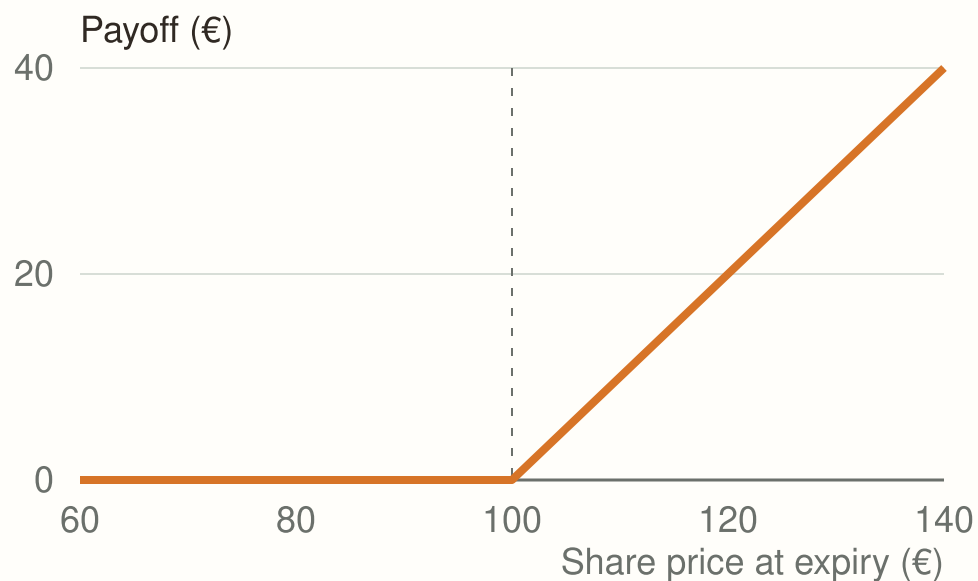
Value: 0

With $K = 100$: a share price of €112 gives a €12 payoff.

S_T : share price at expiry. Payoff excludes the premium paid.

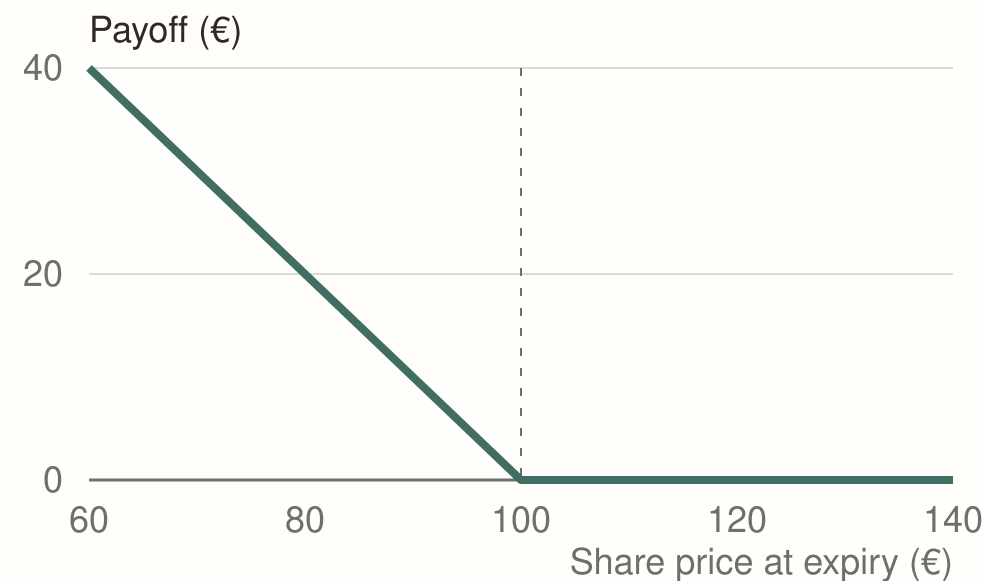
Calls and puts respond in opposite directions.

Call · right to buy



$$C_T = \max(S_T - K, 0)$$

Put · right to sell



$$P_T = \max(K - S_T, 0)$$

Long positions · € per share · strike €100 · payoffs at expiry, before premium

Better terms cannot cost less

ARBITRAGE CONCEPT

#1

Calls

If $K_L < K_H$:

$$C(K_L) \geq C(K_H)$$

The lower strike buys the same asset for less.

Puts

If $K_L < K_H$:

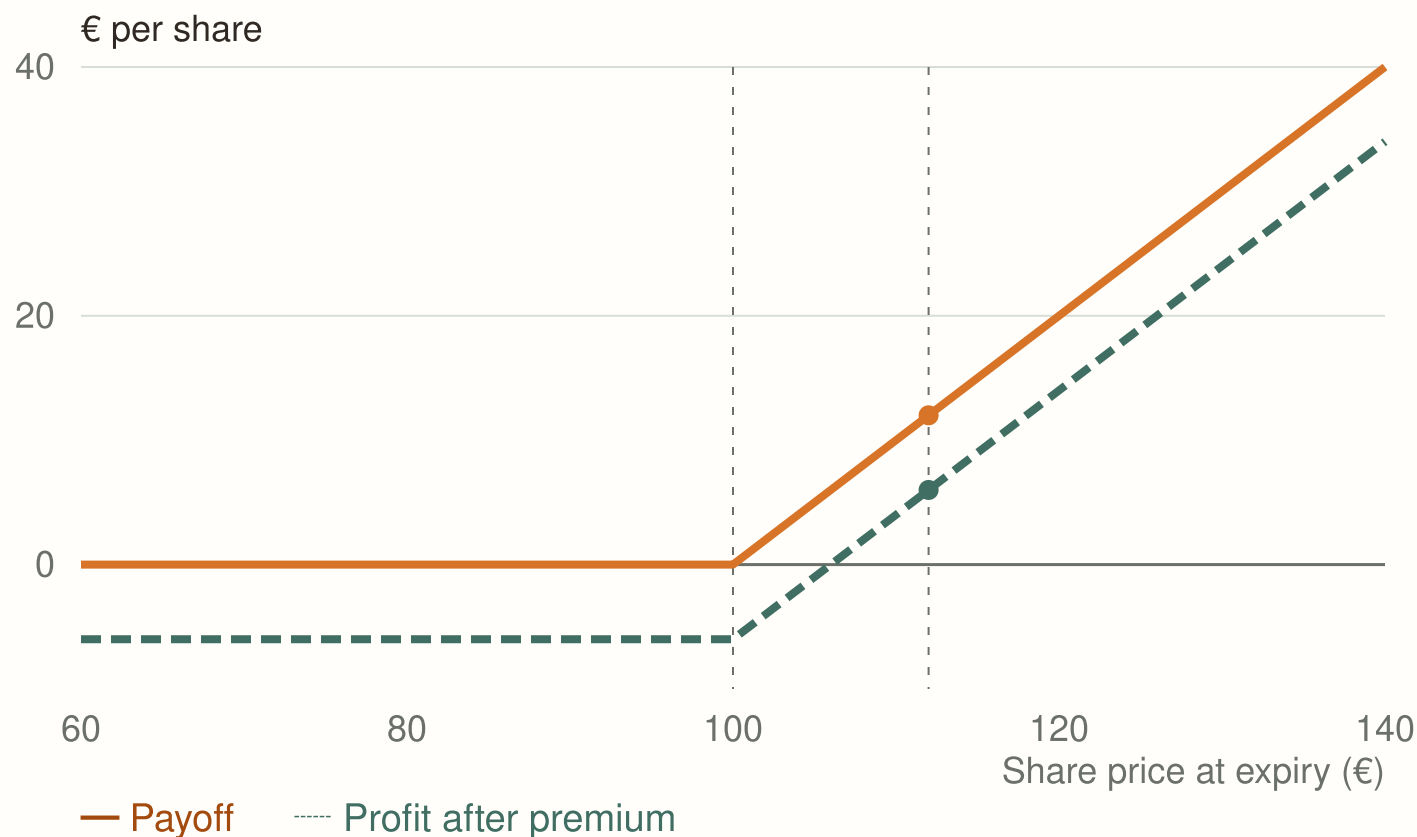
$$P(K_L) \leq P(K_H)$$

The higher strike sells the same asset for more.

If prices violate this ordering, buy the cheaper dominant option and sell the more expensive dominated option.

Same underlying, expiry, exercise style and settlement terms

A positive payoff can still mean a loss.



Share price at expiry **€112**

Premium paid **€6**

Strike **€100**

Payoff **€12**

Profit **€6**

Break-even **€106**

Break-even = strike + premium

Long call · € per share · hold to expiry · ignore financing and transaction costs

Would you exercise, and did you make money?

You paid €6 for a call with strike €100. At expiry, the share is worth €103.

Think it through

1. Is exercise economically worthwhile?
2. What is the payoff?
3. What is the profit?

Yes. €3. –€3.

Exercise value: $103 - 100 = 3$.

Profit: $3 - 6 = -3$.

Exercise recovers value even though the trade lost money.

What about the other side?

So far, we have talked about **holding** an option. But every option holder bought the contract from someone else.

Long · holder

- Pays the premium
- Receives a **right**
- Chooses whether to exercise

Short · writer

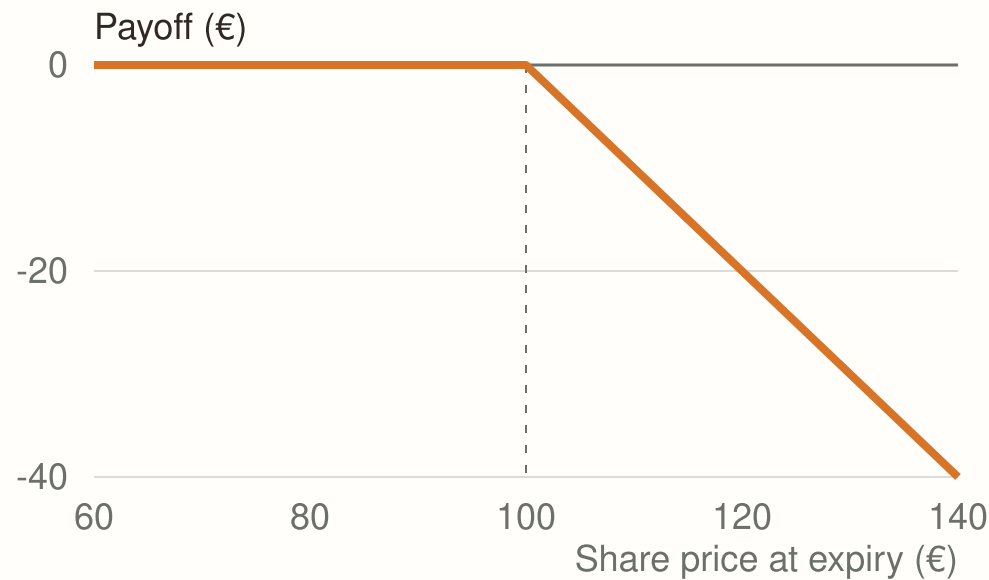
- Receives the premium
- Accepts an **obligation**
- Must perform if assigned

Every long option is matched by a short option.

Before premiums and fees, their payoffs are equal and opposite.

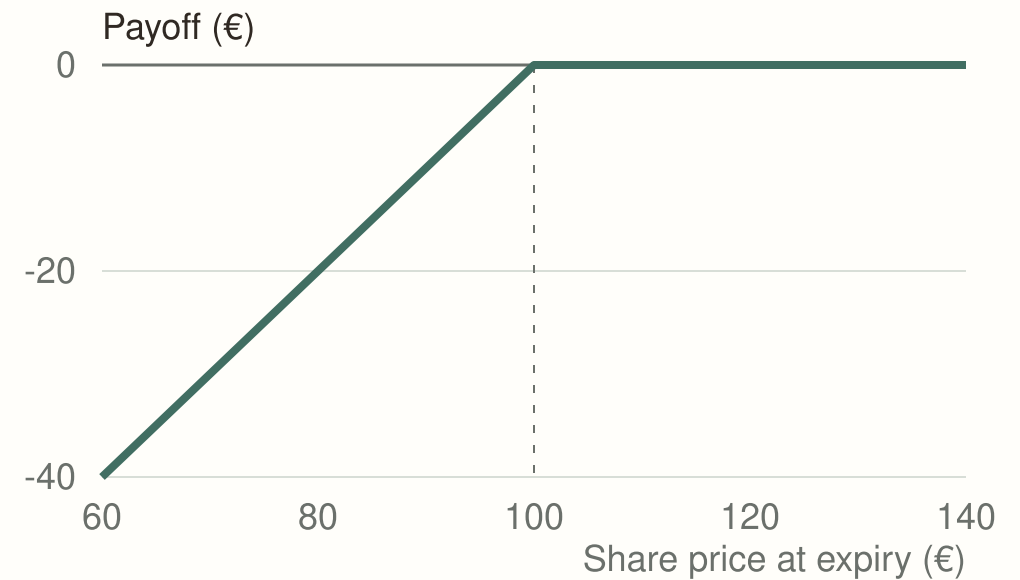
Short calls and puts respond in opposite directions.

Short call · writer



$$C_T^{\text{short}} = -\max(S_T - K, 0)$$

Short put · writer



$$P_T^{\text{short}} = -\max(K - S_T, 0)$$

Short positions · € per share · strike €100 · payoffs at expiry, before premium

What happens when you're short an option?

Assignment connects the holder's exercise decision to a writer's obligation.

1

Exercise

The long holder exercises. At expiry, an automatic procedure may exercise the option.

2

Assignment

The clearing system assigns an open short position; the broker allocates it to a short account.

3

Fulfilment

Short call: sell or deliver at K .

Short put: buy at K .

Once assigned, the writer cannot decline. Cash-settled options fulfil the obligation with a cash payment instead.

The history of options

1600s · Amsterdam

Puts and calls traded on shares of the Dutch East India Company.

1688 · *Confusion of Confusions*

Joseph de la Vega described premiums, exercise, puts and calls.

1900 · Louis Bachelier

Used Brownian motion to derive mathematical option prices in *Théorie de la spéculation*.

1973 · Chicago

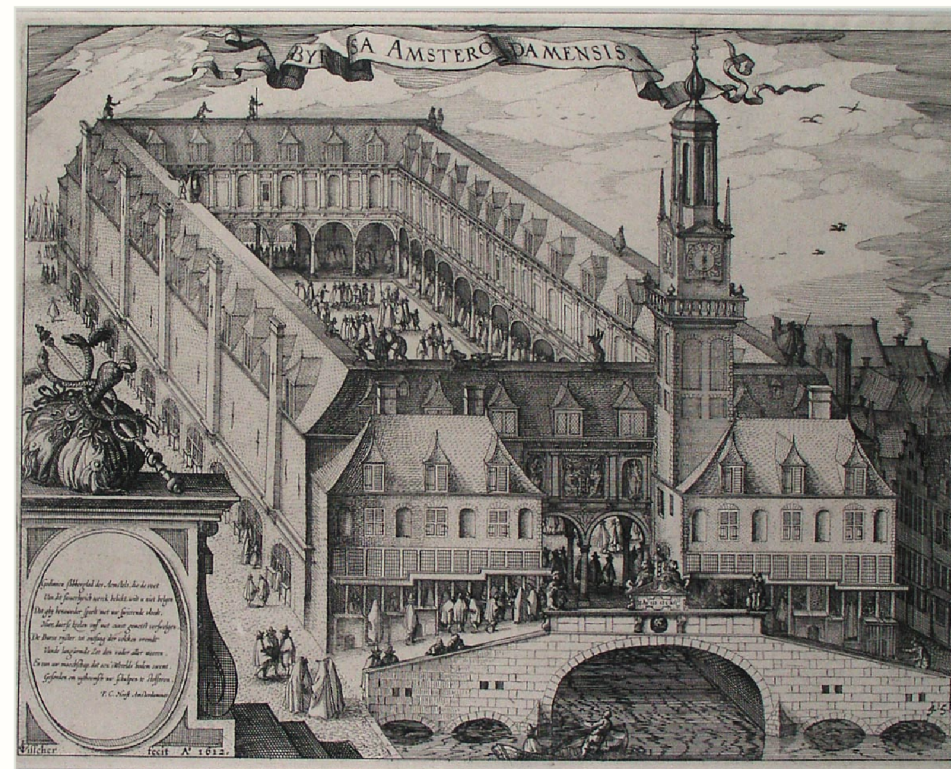
CBOE and its clearing corporation opened a standardized listed market.

1973 · Black–Scholes–Merton

Connected option value to no-arbitrage and dynamic hedging.

2004 · VIX futures

Volatility becomes an asset class.



Amsterdam Stock Exchange, 1612 · Claes Jansz. Visscher · public domain

Options everywhere

Many ordinary contracts have an **option-like payoff**: pay something now to preserve a choice later.

Insurance

Pay a premium; claim only if something bad happens. **Put-like protection.**

Refundable booking

Pay more today; keep the right to cancel and recover value later.

Fixed-rate mortgage

The borrower can often prepay or refinance when rates fall. **A call on the debt.**

Reservation deposit

Pay a small amount now; decide later whether to complete a much larger purchase.

The option may be hidden inside another product. The logic is the same: **right, not obligation.**

Summary

1. An option gives its holder a **right** and its writer an **obligation**.
2. A **call** is the right to buy; a **put** is the right to sell.
3. Exercise depends on the payoff; profit also includes the **premium**.
4. If exercised, an open short position can be **assigned** and must perform.
5. Options have centuries of history.
6. Option-like choices are **everywhere** today.

Next lecture: how can we determine what an option is worth?