Tutorial 7

1 Why did my call lose value?

Imagine that you own a European call option.

(a) All else being equal, the call tends to be worth less as time passes. Explain why.

(b) On one particular day, the price at which you could sell the call falls much more than ordinary time decay would suggest. What might have happened?

2 Which Greek sign combinations are possible?

At zero interest rates and with no dividends, the value of a European call with time to expiry T is

$$C(S,T) = SN(d_1) - KN(d_2), \qquad d_1 = \frac{\ln(S/K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}, \qquad d_2 = d_1 - \sigma\sqrt{T},$$

where N is the standard normal distribution function and $\phi = N'$ is its density. Its Greeks are

$$\Delta = \frac{\partial C}{\partial S} = N(d_1) > 0, \qquad \Gamma = \frac{\partial^2 C}{\partial S^2} = \frac{\phi(d_1)}{S\sigma\sqrt{T}} > 0, \qquad \Theta = -C_T = -\frac{S\phi(d_1)\sigma}{2\sqrt{T}} < 0.$$

For each possible combination of positive or negative delta, gamma, and theta, construct a portfolio of European options if possible and state the conditions under which the construction works. Options at different strikes may have different implied volatilities.

3 Hedged P&L over two days

For a delta-hedged option position, use the approximation

$$\text{Hedged P\&L} \approx \Theta\,\delta t + \frac{1}{2}\Gamma(\delta S)^2 + \nu\,\delta\sigma,$$

where ν is the sensitivity to implied volatility. A call has theta -0.01 dollars per day and gamma 0.03 dollars per squared-dollar move. Over two consecutive days, the stock first rises by one dollar and then falls by one dollar. Assume that the Greeks remain constant, implied volatility does not change, and the hedge is rebalanced daily. Compute the hedged P&L on each day and in total.