Assignment 1: Put-call parity

Lectures 1–3. Show your reasoning and specify the positions you trade, not just the final number. Unless a question says otherwise, options are European, physically settled, and written on one share. Ignore dividends, trading costs, and stock-borrow costs unless specified. All prices are per share.

1 Contracts and payoffs [10 points]

A call and a put have the same strike $K = \$100$ and expiry T. Each costs \$6 today.

(a) State the right held by the buyer of each option and the corresponding obligation of its writer.

(b) For each of $S_T = \$80$, \$100, and \$120, calculate the expiry *payoff* (ignoring the premium) of a long call, long put, short call, and short put. Indicate when the holder exercises.

(c) Calculate the *profit* of the long call and long put in each case. Why can a positive payoff still mean a loss?

2 Uncertainty and better terms [15 points]

Two shares both trade at \$100. At expiry, share A is worth \$100 for certain. Share B is worth either \$80 or \$120, each with pricing probability 1/2. Rates and dividends are zero.

(a) Both shares have the same expected expiry price. Find the value of a strike-\$100 call on each share in this toy model. Explain the difference without appealing only to the formula.

(b) Two otherwise identical calls on one share have strikes \$90 and \$110. Their quoted prices are $C(90) = \$11$ and $C(110) = \$12$. Identify a set of trades which guarantees you a profit, and show why its expiry payoff cannot be negative.

(c) For an option with 25 trading days left, suppose independent daily percentage moves have volatility 1.5% on the first 16 days and 2% on the final 9 days. What is the total move scale (standard deviation) over all 25 days? How many days at a constant 2% daily volatility would give the same total move scale? Show how you combine the two periods.

3 The synthetic share [20 points]

A share costs $S_0 = \$104$. A European call and put have the same strike $K = \$100$ and expiry T. Investing \$98 today pays exactly \$100 at T; borrowing \$98 today requires repaying \$100 at T.

(a) Describe what happens at expiry if you are long the call and short the put. Check both $S_T < K$ and $S_T > K$, including the physical share and strike payment.

(b) What must $C_0 - P_0$ equal if there is no arbitrage? Explain the role of the \$98 rather than just quoting the parity formula.

(c) The call costs \$10 and the put costs \$5. Identify a set of trades which guarantees you a profit. Give the net cash flow today and the net payoff at expiry for $S_T = \$80$ and $S_T = \$120$.

4 Finding arbitrages on an options board [25 points]

The share can be bought or sold for \$100 (a pick'em market). Rates and dividends are zero. The following European options have the same expiry. Each displayed bid and ask is available for ten one-share contracts.

CALLS				STRIKE	PUTS			
Bid size	Bid	Ask	Ask size		Bid size	Bid	Ask	Ask size
(10)	8.00	8.20	(10)	**95**	(10)	2.90	3.10	(10)
(10)	6.20	6.30	(10)	**100**	(10)	6.40	6.50	(10)
(10)	3.20	3.30	(10)	**105**	(10)	8.00	8.10	(10)

(a) For each strike, calculate the parity value of $C - P$. Then determine whether an *executable* parity arbitrage exists. Use bids when selling and asks when buying; do not use midpoints.

(b) For each arbitrage, list the option, share, and cash trades. How much is locked in today per one-share contract? What is the maximum amount for the displayed size of ten contracts?

(c) One row has a midpoint that differs from parity but no executable arbitrage. Identify it and explain why the difference is not free money.

5 Adjustments to parity [15 points]

Treat the following as *independent* scenarios.

(a) A share costs \$110, the strike is \$100, rates and dividends are zero, and $C - P = \$9$. The apparent parity violation requires shorting the share for six months. Borrowing it costs a fixed 2% per year on its current \$110 value, with no other costs. Estimate the borrow cost. Is the apparent \$1 violation still an arbitrage? What is the break-even annual borrow rate?

(b) Now suppose instead that the share costs \$110 and will pay a known dividend worth \$3 today before expiry. The strike is \$100 and rates are zero. For matching European options, what is the adjusted parity value of $C - P$? Explain why the dividend enters with that sign. If $C - P = \$9$, is the call-minus-put portfolio too expensive or too cheap? Would you buy or sell it?

6 Same optionality, different direction [15 points]

Suppose $S_0 = K = \$100$, rates and dividends are zero, and a matching European call and put each cost \$8.

(a) Compute the profit at $S_T = \$80$, \$100, and \$120 of (i) a long put and (ii) a long call combined with a short share initiated at \$100. Show algebraically that their profits agree for *every* nonnegative S_T.

(b) A student says, "A call is a bullish bet, whereas a put is a bearish bet, so they cannot be the same kind of investment." What does the stock trade change? What exposure remains in both long-option positions?

(c) Another student argues that the same expiry-payoff calculation proves *exact* put-call parity for American options under all circumstances. Identify the missing assumption and give one Lecture 3 mechanism that can make this conclusion fail.