Solutions to Assignment 1

1 Contracts and payoffs

A call and a put have the same strike $K = \$100$ and expiry T. Each costs \$6 today. Find the holder's right and writer's obligation, expiry payoffs at $S_T = \$80, \$100, \$120$, and long-option profits.

(a) The call holder may buy one share for \$100; its writer must sell or deliver that share if exercised. The put holder may sell one share for \$100; its writer must buy it if exercised.

(b) Premiums do not enter expiry payoffs:

S_T	Long call	Long put	Short call	Short put
\$80	0	20	0	−20
\$100	0	0	0	0
\$120	20	0	−20	0

The put holder exercises at \$80 and the call holder at \$120. At the strike, exercising gives zero value, so the holder is indifferent.

(c) Subtract the \$6 premium from each long payoff. At $S_T = \$80, \$100, \$120$, long-call profits are respectively $-6, -6, 14$ and long-put profits are $14, -6, -6$. A positive payoff smaller than the premium is still a net loss.

2 Uncertainty and better terms

Shares A and B both trade at \$100. A finishes at \$100 for certain; B finishes at \$80 or \$120 with equal pricing probabilities. Compare strike-\$100 calls. Then consider calls quoted at $C(90) = \$11$ and $C(110) = \$12$, and a 25-day period with daily volatility 1.5% for 16 days and 2% for 9 days.

(a) Both expected expiry prices are \$100. With zero rates, the call on A is worth 0; the call on B is worth

$$\tfrac{1}{2}(80-100)^+ + \tfrac{1}{2}(120-100)^+ = \tfrac{1}{2}(0) + \tfrac{1}{2}(20) = \$10.$$

The call keeps B's upside surprise and discards its downside surprise, so equal means do not imply equal option values.

(b) Buy the $K = 90$ call for \$11 and sell the $K = 110$ call for \$12. Receive \$1 immediately. Their combined expiry payoff is

$$(S_T - 90)^+ - (S_T - 110)^+ = \begin{cases} 0, & S_T \le 90, \\ S_T - 90, & 90 < S_T < 110, \\ 20, & S_T \ge 110. \end{cases}$$

It is never negative; the initial \$1 is guaranteed profit under the stated zero-rate, frictionless assumptions.

(c) Independent daily variances add, even though standard deviations do not:

$$\sigma_{25}^2 = 16(0.015)^2 + 9(0.02)^2 = 0.0072, \qquad \sigma_{25} = \sqrt{0.0072} \approx 0.0849 = 8.49\%.$$

At a constant 2% daily volatility, $n(0.02)^2 = 0.0072$, hence $n = 18$ trading days. The assumption is independent daily moves with the stated daily variances.

3 The synthetic share

The share costs \$104; matching European options have strike \$100. An investment of \$98 today pays \$100 at expiry. The call costs \$10 and the put costs \$5.

(a) Above K, exercise the long call: pay \$100 and receive a share. The short put expires. Below K, the call expires and the short put is assigned: pay \$100 and receive a share. Thus long call plus short put gives one share in exchange for \$100 at expiry in either case.

(b) The options together cost $C_0 - P_0$. Buying a share for \$104 and borrowing \$98 replicates that same expiry outcome: a share now, and a \$100 cash liability at T. Therefore

$$C_0 - P_0 = S_0 - \mathrm{PV}(K) = 104 - 98 = \$6.$$

(c) The quoted option portfolio costs $10 - 5 = \$5$, so it is \$1 too cheap. Buy the call (-10), sell the put ($+5$), short one borrowed share ($+104$), and invest \$98 (-98). Today's net cash flow is $+1$.

At $S_T = \$80$, put assignment buys a share for \$100; at $S_T = \$120$, call exercise buys a share for \$100. In both cases, the investment supplies the \$100, and the acquired share closes the stock short. The expiry net is 0, so the initial \$1 is guaranteed.

4 Finding arbitrages on an options board

The stock is a \$100 pick'em market, with zero rates and dividends. All option bids and asks below are executable for ten one-share contracts.

CALLS				STRIKE	PUTS			
Bid size	Bid	Ask	Ask size		Bid size	Bid	Ask	Ask size
(10)	8.00	8.20	(10)	**95**	(10)	2.90	3.10	(10)
(10)	6.20	6.30	(10)	**100**	(10)	6.40	6.50	(10)
(10)	3.20	3.30	(10)	**105**	(10)	8.00	8.10	(10)

(a) Parity requires $C - P = S_0 - K$, giving $5, 0, -5$ at strikes $95, 100, 105$. With spreads, the executable no-arbitrage test is

$$C_{\text{bid}} - P_{\text{ask}} \le S_0 - K \le C_{\text{ask}} - P_{\text{bid}}.$$

The quoted intervals are $[4.90, 5.30]$, $[-0.30, -0.10]$, and $[-4.90, -4.70]$. Only the 95 row contains its parity value.

(b) At $K = 100$, the synthetic share is cheap: buy the call at 6.30, sell the put at 6.40, short the share at 100, and invest 100. The option pair brings in 0.10 today, and the other two trades offset. At expiry the options deliver a share for 100; the invested cash pays for it and it closes the short. Profit: 0.10 per contract, or \$1 for ten.

At $K = 105$, the synthetic share is expensive: sell the call at 3.20, buy the put at 8.10, buy the share for 100, and borrow 105 to repay at expiry. Today's net is $3.20 - 8.10 - 100 + 105 = \$0.10$. At expiry, share plus short call plus long put yields 105 whatever S_T; that repays the loan. Profit: 0.10 per contract, or \$1 for ten.

(c) At $K = 95$, call midpoint minus put midpoint is $8.10 - 3.00 = 5.10$, not 5. But selling the pair yields only $8.00 - 3.10 = 4.90$, while buying it costs $8.20 - 2.90 = 5.30$. Neither executable side crosses the parity value.

5 Adjustments to parity

In separate scenarios, examine (a) a \$110 share with $K = \$100$, $C - P = \$9$, and a fixed 2% annual stock-borrow rate for six months, and (b) a \$110 share paying a known dividend worth \$3 today, again with $K = \$100$ and $C - P = \$9$. Rates are zero.

(a) Clean parity gives $C - P = 110 - 100 = \$10$, so the quote appears \$1 too low. The trade needs a short share, whose fixed borrow cost is approximately

$$B_0 = 110(0.02)(0.5) = \$1.10.$$

The net result would be $1 - 1.10 = -\$0.10$, not an arbitrage. The borrow-adjusted lower bound is $C - P \geq 110 - 100 - 1.10 = \$8.90$, and the quote of \$9 respects it. The break-even annual rate solves $110b(0.5) = 1$, so $b = 1/55 \approx 1.82\%$.

(b) The stock owner receives the dividend; the call-minus-put portfolio does not receive it before expiry. Thus

$$C - P = S_0 - D_0 - K = 110 - 3 - 100 = \$7.$$

At \$9 the option portfolio is \$2 too expensive, so sell it: short the call and buy the put. One matching hedge is to buy the share and borrow \$100: its net cost with the option sale is $110 - 100 - 9 = \$1$ today, it receives the \$3 dividend, and the expiry positions cancel. The net gain is \$2.

6 Same optionality, different direction

The stock and strike are both \$100, rates and dividends are zero, and matching European call and put premiums are both \$8. Compare a long put with a long call plus a short share. Then assess a claim of exact parity for American options.

(a) Both strategies have the same profit:

S_T	Long put	Long call + short share
\$80	$20 - 8 = 12$	$0 - 8 + (100 - 80) = 12$
\$100	$0 - 8 = -8$	$0 - 8 + (100 - 100) = -8$
\$120	$0 - 8 = -8$	$20 - 8 + (100 - 120) = -8$

Generally,

$$(S_T - 100)^+ - 8 + (100 - S_T) = (100 - S_T)^+ - 8,$$

using $x^+ - x = (-x)^+$ with $x = S_T - 100$.

(b) The share trade changes the position's linear directional exposure: it turns the call's upward-sloping side into the put's downward-sloping side. The kink remains. Both positions are long optionality and benefit from sufficiently favorable distributional outcomes relative to their premium, not merely from a change in the mean.

(c) Exact European parity assumes both options are held to the same fixed expiry, so their cash flows match at that date. American exercise can move cash and share delivery to an earlier date. A dividend may make early call exercise valuable; positive interest can make early put exercise valuable. The common expiry-payoff identity therefore does not prove exact American price parity in all circumstances.